Much of this is taken almost directly from both the Stata manual on ivregress and lecture notes from Sally Stearn (UNC). We are interested in estimating the effect of housing value on rent, however as the Stata manual states we are worried about random shocks (our omitted variable) affecting both housing values and rent. To counteract this, we use average family income and census region as instruments for housing value. The data is state level averages from 1980.

```
. use https://www.stata-press.com/data/r18/hsng
(1980 Census housing data)

. sum

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
       state |          0
    division |         50        5.12    2.560612          1          9
      region |         50        2.66    1.061574          1          4
         pop |         50     4518149     4715038     401851   2.37e+07
     popgrow |         50       16.29    14.38729       -3.6       63.8
-------------+---------------------------------------------------------
      popden |         50     1543.72    2213.894          7       9862
    pcturban |         50    66.94913    14.40956   33.77319   91.29498
      faminc |         50    19499.92    2617.218      14591      28395
        hsng |         50     1762686     1847308     162825    9279036
    hsnggrow |         50    35.65202    19.30491   9.015595   97.00565
-------------+---------------------------------------------------------
     hsngval |         50       48484    15770.24      31100     119400
        rent |         50      234.76    35.35335        180        368
        reg1 |         50         .18    .3880879          0          1
        reg2 |         50         .24    .4314191          0          1
        reg3 |         50         .32    .4712121          0          1
-------------+---------------------------------------------------------
        reg4 |         50         .26    .4430875          0          1
```

Start with the OLS regression (we are worried that this might be biased due to random shocks).

```
. reg rent hsngval pcturban

      Source |       SS           df       MS      Number of obs   =        50
-------------+----------------------------------   F(2, 47)        =     47.54
       Model |  40983.5269         2  20491.7635   Prob > F        =    0.0000
    Residual |  20259.5931        47  431.055172   R-squared       =    0.6692
-------------+----------------------------------   Adj R-squared   =    0.6551
       Total |    61243.12        49  1249.85959   Root MSE        =    20.762

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0015205   .0002276     6.68   0.000     .0010627    .0019784
    pcturban |   .5248216   .2490782     2.11   0.040     .0237408    1.025902
       _cons |   125.9033   14.18537     8.88   0.000     97.36603    154.4406
------------------------------------------------------------------------------
```

Store the estimates to be used later in the Durban-Wu-Hausman test.

```
. est store my_OLS
```

Before we run our IV, we will test the power of the instrument by manually running the first stage. Note that we include the instruments as well as any control variables that will also be included in the second stage.

```
. reg hsngval faminc reg2-reg4 pcturban

      Source |       SS           df       MS      Number of obs   =        50
-------------+----------------------------------   F(5, 44)        =     19.66
       Model |  8.4187e+09         5  1.6837e+09   Prob > F        =    0.0000
    Residual |  3.7676e+09        44  85626930.6   R-squared       =    0.6908
-------------+----------------------------------   Adj R-squared   =    0.6557
       Total |  1.2186e+10        49   248700555   Root MSE        =    9253.5

------------------------------------------------------------------------------
     hsngval |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
```

```
-------------+----------------------------------------------------------------
      faminc |   2.731324   .6818931     4.01   0.000     1.357058    4.105589
        reg2 |  -5095.038   4122.112    -1.24   0.223    -13402.61    3212.533
        reg3 |   -1778.05   4072.691    -0.44   0.665    -9986.019    6429.919
        reg4 |   13413.79   4048.141     3.31   0.002     5255.296    21572.28
    pcturban |   182.2201   115.0167     1.58   0.120    -49.58092    414.0211
       _cons |  -18671.87   11995.48    -1.56   0.127    -42847.17    5503.439
------------------------------------------------------------------------------
```

The F-test can be run after the first stage has been estimated. The F-statistic is above 10, which is thought of by many as the minimum threshold to not worry about a weak instrument problem.

```
. testparm faminc reg2-reg4

 ( 1)  faminc = 0
 ( 2)  reg2 = 0
 ( 3)  reg3 = 0
 ( 4)  reg4 = 0

       F(  4,    44) =   13.30
            Prob > F =    0.0000
```

Next we estimate the IV. Note that we are using two stage least squares (2sls). The outcome in the second stage is included first, then any control variables that will be included in both the second and first stage, with the first stage in the parenthesis. The endogenous variable is included to the left of the = and the instruments that will be omitted from the second stage are included to the right. I have also added the "first" option that reports the results of the first stage in the output.

```
. ivregress 2sls rent pcturban (hsngval = faminc reg2-reg4), first

First-stage regressions
-----------------------

                                                Number of obs     =         50
                                                F(   5,     44)   =      19.66
                                                Prob > F          =     0.0000
                                                R-squared         =     0.6908
                                                Adj R-squared     =     0.6557
                                                Root MSE          =  9253.4821

------------------------------------------------------------------------------
     hsngval |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    pcturban |   182.2201   115.0167     1.58   0.120    -49.58092    414.0211
      faminc |   2.731324   .6818931     4.01   0.000     1.357058    4.105589
        reg2 |  -5095.038   4122.112    -1.24   0.223    -13402.61    3212.533
        reg3 |   -1778.05   4072.691    -0.44   0.665    -9986.019    6429.919
        reg4 |   13413.79   4048.141     3.31   0.002     5255.296    21572.28
       _cons |  -18671.87   11995.48    -1.56   0.127    -42847.17    5503.439
------------------------------------------------------------------------------


Instrumental variables (2SLS) regression          Number of obs     =         50
                                                  Wald chi2(2)      =      90.76
                                                  Prob > chi2       =     0.0000
                                                  R-squared         =     0.5989
                                                  Root MSE          =     22.166

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0022398   .0003284     6.82   0.000     .0015961    .0028836
    pcturban |    .081516   .2987652     0.27   0.785    -.504053     .667085
       _cons |   120.7065   15.22839     7.93   0.000     90.85942    150.5536
------------------------------------------------------------------------------
Instrumented:  hsngval
Instruments:   pcturban faminc reg2 reg3 reg4
```

Store the estimates to use later in the Durban-Wu-Hausman test.

```
. est store my_IV
```

After running the IV you can use this command to estimate several first stage weak instrument tests. Note that the F is the same as the F-test we conducted above. We also have the partial R2 which is another common test for the strength of an instrument. In this case 0.547 is quite large. Rather than using the critical value of 10 for the F-test, the output includes some different critical values depending on the amount of relative bias that the researcher is willing to accept.

```
. estat firststage

  First-stage regression summary statistics
  --------------------------------------------------------------------------
               |            Adjusted      Partial
      Variable |   R-sq.       R-sq.        R-sq.       F(4,44)   Prob > F
  -------------+------------------------------------------------------------
       hsngval |  0.6908      0.6557       0.5473       13.2978    0.0000
  --------------------------------------------------------------------------


  Minimum eigenvalue statistic = 13.2978

  Critical Values                      # of endogenous regressors:    1
  Ho: Instruments are weak             # of excluded instruments:     4
  ---------------------------------------------------------------------
                                     |    5%     10%     20%     30%
  2SLS relative bias                 |  16.85   10.27    6.71    5.34
  -----------------------------------+---------------------------------
                                     |   10%     15%     20%     25%
  2SLS Size of nominal 5% Wald test  |  24.58   13.96   10.26    8.31
  LIML Size of nominal 5% Wald test  |   5.44    3.87    3.30    2.98
  ---------------------------------------------------------------------
```

The Durban-Wu-Hausman tests if the coefficients from a consistent estimator (IV) are different from those estimated from a more efficient but potentially biased estimator (OLS). The Chi2 indicates that the coefficients are different, meaning X is endogenous and we should be using IV. However, there is a common problem with this test where the V_b-V_B is not positive definite. I want to reiterate that to conduct this test and the variants of the test, we need to assume that the instrument is strong and not correlated with the error term – so this is NOT a test of an instrument's suitability.

```
. hausman my_IV my_OLS, sigmamore

Note: the rank of the differenced variance matrix (1) does not equal the number of
        coefficients being tested (2); be sure this is what you expect, or there
        may be problems computing the test.  Examine the output of your estimators
        for anything unexpected and possibly consider scaling your variables so
        that the coefficients are on a similar scale.

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     my_IV        my_OLS       Difference          S.E.
-------------+----------------------------------------------------------------
     hsngval |    .0022398     .0015205        .0007193         .000207
    pcturban |     .081516     .5248216       -.4433056        .1275655
------------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from ivregress
          B = inconsistent under Ha, efficient under Ho; obtained from regress

    Test:  Ho:  difference in coefficients not systematic

                  chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =       12.08
                Prob>chi2 =      0.0005
                (V_b-V_B is not positive definite)
```

There are a few variants of the Hausman test. The first is to predict the endogenous variable using the first stage regression and include in the second stage regression. The second is to include the residual instead. I calculate both the predicted X and residual below.

```
. reg hsngval faminc reg2-reg4 pcturban

      Source |       SS           df       MS      Number of obs   =        50
-------------+----------------------------------   F(5, 44)        =     19.66
       Model |  8.4187e+09         5  1.6837e+09   Prob > F        =    0.0000
    Residual |  3.7676e+09        44  85626930.6   R-squared       =    0.6908
-------------+----------------------------------   Adj R-squared   =    0.6557
       Total |  1.2186e+10        49   248700555   Root MSE        =    9253.5

------------------------------------------------------------------------------
     hsngval |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      faminc |   2.731324   .6818931     4.01   0.000     1.357058    4.105589
        reg2 |  -5095.038   4122.112    -1.24   0.223    -13402.61    3212.533
        reg3 |   -1778.05   4072.691    -0.44   0.665    -9986.019    6429.919
        reg4 |   13413.79   4048.141     3.31   0.002     5255.296    21572.28
    pcturban |   182.2201   115.0167     1.58   0.120    -49.58092    414.0211
       _cons |  -18671.87   11995.48    -1.56   0.127    -42847.17    5503.439
------------------------------------------------------------------------------
```

Predicted X

```
. predict hsng_hat
(option xb assumed; fitted values)
```

Residual

```
. predict FSres, res

. su hsngval hsng_hat FSres

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
     hsngval |         50       48484    15770.24      31100     119400
    hsng_hat |         50       48484    13107.67   28025.76   84022.63
       FSres |         50    .0000372    8768.665  -10503.84   46755.89
```

Before conducting those tests, I want to show that we get the same coefficient as IV when we estimate the second stage using the predicted X from the first stage, however our standard errors are not correct. Compare this…

```
. reg rent hsng_hat pcturban

      Source |       SS           df       MS      Number of obs   =        50
-------------+----------------------------------   F(2, 47)        =     62.94
       Model |  44592.8127         2  22296.4063   Prob > F        =    0.0000
    Residual |  16650.3073        47  354.261858   R-squared       =    0.7281
-------------+----------------------------------   Adj R-squared   =    0.7166
       Total |    61243.12        49  1249.85959   Root MSE        =    18.822

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    hsng_hat |   .0022398   .0002789     8.03   0.000     .0016788    .0028009
    pcturban |   .0815159   .2536953     0.32   0.749    -.4288532    .5918851
       _cons |   120.7065   12.93113     9.33   0.000     94.69244    146.7206
------------------------------------------------------------------------------
```

To this.

```
. ivregress 2sls rent pcturban (hsngval = faminc reg2-reg4)

Instrumental variables (2SLS) regression         Number of obs   =        50
                                                 Wald chi2(2)    =     90.76
                                                 Prob > chi2     =    0.0000
                                                 R-squared       =    0.5989
                                                 Root MSE        =    22.166

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0022398   .0003284     6.82   0.000     .0015961    .0028836
    pcturban |    .081516   .2987652     0.27   0.785     -.504053     .667085
       _cons |   120.7065   15.22839     7.93   0.000     90.85942    150.5536
```

```
------------------------------------------------------------------------------
Instrumented:  hsngval
Instruments:   pcturban faminc reg2 reg3 reg4
```

Now back to our first test, estimate the second stage including the X and predicted X. The null is that the predicted X should not provide different variation from X (or that the coefficient is 0) and the alternative is that it does. We should only get a different value if the cov(X,e) does not equal 0 (or X is endogenous). In this case we reject the null that there is no different coefficient (see the t-stat on the hsng_hat variable).

```
. reg rent hsngval hsng_hat pcturban

      Source |       SS           df       MS      Number of obs   =        50
-------------+----------------------------------   F(3, 46)        =     47.05
       Model |   46189.152         3   15396.384   Prob > F        =    0.0000
    Residual |   15053.968        46  327.260173   R-squared       =    0.7542
-------------+----------------------------------   Adj R-squared   =    0.7382
       Total |    61243.12        49  1249.85959   Root MSE        =     18.09

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0006509   .0002947     2.21   0.032     .0000577    .0012442
    hsng_hat |   .0015889   .0003984     3.99   0.000      .000787    .0023908
    pcturban |   .0815159   .2438355     0.33   0.740    -.4092994    .5723313
       _cons |   120.7065   12.42856     9.71   0.000     95.68912    145.7239
------------------------------------------------------------------------------
```

You come to the same conclusion using the residual from the first stage.

```
. reg rent hsngval FSres pcturban

      Source |       SS           df       MS      Number of obs   =        50
-------------+----------------------------------   F(3, 46)        =     47.05
       Model |  46189.1513         3  15396.3838   Prob > F        =    0.0000
    Residual |  15053.9687        46   327.26019   R-squared       =    0.7542
-------------+----------------------------------   Adj R-squared   =    0.7382
       Total |    61243.12        49  1249.85959   Root MSE        =     18.09

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0022398   .0002681     8.36   0.000     .0017003    .0027794
       FSres |  -.0015889   .0003984    -3.99   0.000    -.0023908    -.000787
    pcturban |    .081516   .2438355     0.33   0.740    -.4092994    .5723313
       _cons |   120.7065   12.42856     9.71   0.000     95.68912    145.7239
------------------------------------------------------------------------------
```

The last variant is to include the instruments in the second stage. The null here is that the instruments should have a 0 coefficient. We find that is not the case. Note that I used an F-test since we have multiple instruments.

```
. reg rent hsngval faminc reg2-reg4 pcturban

      Source |       SS           df       MS      Number of obs   =        50
-------------+----------------------------------   F(6, 43)        =     38.99
       Model |  51734.9429         6  8622.49049   Prob > F        =    0.0000
    Residual |  9508.17708        43  221.120397   R-squared       =    0.8447
-------------+----------------------------------   Adj R-squared   =    0.8231
       Total |    61243.12        49  1249.85959   Root MSE        =     14.87

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0006509   .0002423     2.69   0.010     .0001624    .0011395
      faminc |   .0087876   .0012801     6.86   0.000     .0062061    .0113691
        reg2 |  -14.86383   6.738152    -2.21   0.033    -28.45261   -1.275052
        reg3 |   2.022185   6.558874     0.31   0.759    -11.20504    15.24942
        reg4 |   2.544457   7.271764     0.35   0.728    -12.12045    17.20937
    pcturban |   .0882531   .1900277     0.46   0.645    -.2949744    .4714805
       _cons |   28.19295   19.80007     1.42   0.162    -11.73771     68.1236
```

```
------------------------------------------------------------------------------

. testparm faminc reg2-reg4

 ( 1)  faminc = 0
 ( 2)  reg2 = 0
 ( 3)  reg3 = 0
 ( 4)  reg4 = 0

       F(  4,    43) =   12.16
            Prob > F =    0.0000
```

We can run some tests to determine if the exclusion restriction hold for our instruments if we have multiple instruments and assume that at least one of them is legitimate. The first test we will conduct is the LM test (or Sargan test). First estimate the IV.

```
. ivregress 2sls rent pcturban (hsngval = faminc reg2-reg4)

Instrumental variables (2SLS) regression          Number of obs   =         50
                                                  Wald chi2(2)    =      90.76
                                                  Prob > chi2     =     0.0000
                                                  R-squared       =     0.5989
                                                  Root MSE        =     22.166

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0022398   .0003284     6.82   0.000     .0015961    .0028836
    pcturban |    .081516   .2987652     0.27   0.785     -.504053     .667085
       _cons |   120.7065   15.22839     7.93   0.000     90.85942    150.5536
------------------------------------------------------------------------------
Instrumented:  hsngval
Instruments:   pcturban faminc reg2 reg3 reg4
```

Predict the residual from the IV.

```
. predict IVRES, res
```

Regress the residual on the controls and instruments.

```
. reg IVRES pcturban faminc reg2-reg4

      Source |       SS           df       MS      Number of obs   =        50
-------------+----------------------------------   F(5, 44)        =      2.57
       Model |  5545.79184         5  1109.15837   Prob > F        =    0.0402
    Residual |  19019.9253        44  432.271031   R-squared       =    0.2258
-------------+----------------------------------   Adj R-squared   =    0.1378
       Total |  24565.7172        49  501.341167   Root MSE        =    20.791

------------------------------------------------------------------------------
       IVRES |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    pcturban |  -.2827938   .2584246    -1.09   0.280    -.8036144    .2380269
      faminc |   .0044478   .0015321     2.90   0.006       .00136    .0075356
        reg2 |  -6.768287   9.261742    -0.73   0.469     -25.4341    11.89753
        reg3 |   4.847343     9.1507     0.53   0.599    -13.59468    23.28937
        reg4 |  -18.76881    9.09554    -2.06   0.045    -37.09967   -.4379549
       _cons |   -62.8457   26.95197    -2.33   0.024    -117.1638    -8.52757
------------------------------------------------------------------------------
```

We need to calculate N*R2. Typing this command shows you all the results that are being held in temporary memory.

```
. ereturn list

scalars:
                  e(N) =  50
               e(df_m) =  5
               e(df_r) =  44
                  e(F) =  2.565886421248727
                 e(r2) =  .2257533047709994
```

```
               e(rmse) =  20.79112865231076
                e(mss) =  5545.791839052552
                e(rss) =  19019.92534802524
               e(r2_a) =  .1377707257677039
                 e(ll) =  -219.4774118038267
               e(ll_0) =  -225.8740300160578
               e(rank) =  6

macros:
            e(cmdline) : "regress IVRES pcturban faminc reg2-reg4"
              e(title) : "Linear regression"
          e(marginsok) : "XB default"
                e(vce) : "ols"
             e(depvar) : "IVRES"
                e(cmd) : "regress"
         e(properties) : "b V"
            e(predict) : "regres_p"
              e(model) : "ols"
          e(estat_cmd) : "regress_estat"

matrices:
                  e(b) :  1 x 6
                  e(V) :  6 x 6

functions:
             e(sample)
```

Here is our N*R2

```
. scalar nr2 = e(N)*e(r2)

. di nr2
11.287665
```

And the test statistic is distributed Chi-2 with degrees of freedom = # of overidentified instruments (in this case we have 4 instruments or 3 extra). The null is that there is no relationship between the residuals and the instruments, which in this case we reject. This means at least one of those instruments is bad.

```
. di 1 - chi2(3, nr2)
.01026785
```

A simpler way to do this is to estimate the IV.

```
. ivregress 2sls rent pcturban (hsngval = faminc reg2-reg4)

Instrumental variables (2SLS) regression          Number of obs   =         50
                                                  Wald chi2(2)    =      90.76
                                                  Prob > chi2     =     0.0000
                                                  R-squared       =     0.5989
                                                  Root MSE        =     22.166

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0022398   .0003284     6.82   0.000     .0015961    .0028836
    pcturban |    .081516   .2987652     0.27   0.785     -.504053     .667085
       _cons |   120.7065   15.22839     7.93   0.000     90.85942    150.5536
------------------------------------------------------------------------------
Instrumented:  hsngval
Instruments:   pcturban faminc reg2 reg3 reg4
```

Then use the estat overid command. The Sargan score is the LM test we just calculated.

```
. estat overid

  Tests of overidentifying restrictions:

  Sargan (score) chi2(3) =  11.2877  (p = 0.0103)
  Basmann chi2(3)        =  12.8294  (p = 0.0050)
```

We know that at least one of the instruments is problematic. To do this next test (another Hausman) we need to first make an assumption about which instrument we believe is legitimate and then test if the others are problematic. In this example I assumed that the family income instrument is legitimate. So, estimate the IV just using family income as an instrument.

```
. ivregress 2sls rent pcturban (hsngval = faminc)

Instrumental variables (2SLS) regression         Number of obs   =         50
                                                 Wald chi2(2)    =      51.44
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.2887
                                                 Root MSE        =     29.517

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     hsngval |   .0031938   .0006206     5.15   0.000     .0019775    .0044101
    pcturban |  -.5064118   .4815558    -1.05   0.293    -1.450244    .4374202
       _cons |   113.8143   20.52667     5.54   0.000      73.5828    154.0459
------------------------------------------------------------------------------
Instrumented:  hsngval
Instruments:   pcturban faminc

. est store my_IVe
```

And then compare the coefficients from the IV using only family income (this is our consistent estimator) to IV using all the instruments (this is our more efficient estimator that might be biased). The null is that there is no difference in the coefficients and we can use the full IV. The alternative is that the coefficients are different and we should only use family income because the extra instruments are problematic (correlated with the error). In this case we reject the null and conclude that we should not use extra instruments (census region). Just to reiterate, we have to assume that family income is a legitimate instrument to conduct this test.

```
. hausman my_IVe my_IV, sigmamore

Note: the rank of the differenced variance matrix (1) does not equal the number of
        coefficients being tested (2); be sure this is what you expect, or there
        may be problems computing the test.  Examine the output of your estimators
        for anything unexpected and possibly consider scaling your variables so
        that the coefficients are on a similar scale.

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     my_IVe       my_IV        Difference          S.E.
-------------+----------------------------------------------------------------
     hsngval |    .0031938     .0022398         .000954        .0003306
    pcturban |   -.5064118      .081516       -.5879278        .2037461
------------------------------------------------------------------------------
                       b = consistent under Ho and Ha; obtained from ivregress
        B = inconsistent under Ha, efficient under Ho; obtained from ivregress

    Test:  Ho:  difference in coefficients not systematic

                  chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =        8.33
                Prob>chi2 =      0.0039

. log close
```