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MEASURING UP: HOW THE HIGHEST PERFORMING STATE (MASSACHUSETTS) COMPARES TO THE HIGHEST PERFORMING COUNTRY (HONG KONG) IN GRADE 3 MATHEMATICS

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EXECUTIVE SUMMARY

Hong Kong ranked highest among all countries on the Grade 4 mathematics component of the 2007 Trends in Mathematics and Science Study (TIMSS-4). Massachusetts, the highest scoring state on the U.S. National Assessment of Educational Progress (NAEP), ranked a respectable fourth internationally on TIMSS-4, where it participated as a country in the benchmarking component of the 2007 TIMSS-4, but still scored considerably below Hong Kong. To help understand why Hong Kong students outperform Massachusetts students on the common TIMSS international assessment, this study identifies differences between the items on Hong Kong's and Massachusetts' internal mathematics assessments administered in the spring of Grade 3 to gather insight into the relative mathematical expectations in Hong Kong and Massachusetts.[1]

The Hong Kong performance advantage over Massachusetts was especially large in the percentage of its students achieving at the very highest performance levels. For example, 40% of Hong Kong students achieved at the advanced TIMSS level, compared with only 22% of Massachusetts students. Our hypothesis is that exposure, in Hong Kong, to mathematics problems that are computationally more difficult and/or cognitively more complex than those in Massachusetts at the same, or in an earlier, grade contributes to Hong Kong's advantage in the high-performance range.

Hence, identifying differences between the Hong Kong and Massachusetts internal mathematics assessment items may guide Massachusetts and lower-performing U.S. states to reexamine and strengthen their mathematics assessments. Moreover, focusing on mathematical expectations related to these observed item differences in the early grades is important given that research shows a strong correlation between a country's initial performance on international assessments in the early grades and its later performance in the upper grades (Ginsburg, Leinwand, Cooke, Noell, & Pollock, 2005).

The Hong Kong assessment is a basic competency assessment used as a formative assessment to aid schools in evaluating and strengthening their mathematics performance. Scores for individual students are not reported to schools. In contrast, the Massachusetts assessment is a high-stakes assessment used to gauge adequate yearly progress, as defined at state levels, for the reporting associated with the No Child Left Behind Act, although this assessment also may be used formatively. A purely formative assessment may allow greater room to assess the full range of mathematical abilities than an assessment that is also high stakes, given that the emphasis in the latter is on measuring whether lower-performing students achieve adequate performance.

Items on each assessment were classified along several dimensions:

- Item *strand* identified whether an item addresses numbers, measurement, geometry, data and probability, or algebra.
- Item *type* differentiated multiple-choice items, for which a student selects a response, and constructed-response items, for which the student generates the response.

[1] Hong Kong tests students only in Grades 3, 6, and 9. Grade 3 was chosen for the comparison as the grade closest to the TIMSS Grade 4 results.

Constructed-response items were further identified as either short closed constructed-response or as open constructed-response.

- Item *computational difficulty* rated items as low, medium, or high on the basis of such considerations as the number of digits in the operations, whether regrouping was required, or the presence of medial zeros in subtraction and division, as well as on the basis of typical curricular expectations for U.S. third graders.
- Item *cognitive complexity* used a three-level rating of Level 1 (low), Level 2 (moderate), and Level 3 (high), where low required only knowing basic facts and procedures; moderate required applying knowledge and procedures to solve routine problems; and high required reasoning in nonroutine situations.

The comparisons of the two assessments found that Hong Kong items differed from Massachusetts items in several important ways:

1. *Hong Kong items were more concentrated in the number and measurement strands (75%), compared with Massachusetts items (60%).* A firm understanding of basic number concepts is essential for doing more-advanced work in fractions and algebra. Further, a solid understanding of measurement topics—length, volume, money, weight, and maps—is critical to handling real-world mathematics applications, including later work in geometry. The Hong Kong assessment includes no items under algebra, and the Massachusetts assessment has just over 10% of its items under algebra—although Massachusetts' own classification framework identifies 20% of the items as "patterns, relations and algebra." The U.S. National Mathematics Panel has discouraged overly stressing algebra, especially patterns, in the early grades. Hong Kong items were also less concentrated on data and probability (6%) compared with Massachusetts items (17%), but this emphasis on data and probability in Massachusetts may be justified because it represents an area of relative performance strength in Massachusetts.

2. *Hong Kong items were more likely to require students to construct a response (86%) than Massachusetts items (29%).* Constructed-response items tend to be more demanding in that students must generate the correct answer by working completely through the problem without the advantage of being able to select a correct answer from a list.

3. *Hong Kong items were more likely to require more than low computational difficulty (37%), compared with Massachusetts items (3%).* In the numbers domain, where computation is an integral component of the solution, 13 out of 15 (87%) of Hong Kong items were of higher computational difficulty, whereas only 1 out of 17 (6%) of the Massachusetts items in numbers required more than simple computational skills. Notably, a study of high-performing countries on TIMSS and the Programme for International Students Assessment (PISA), another international assessment, found that computational skills were associated with their success. Countries with high performance on items that were cognitively demanding were also high performers on procedural computational problems (Ginsburg, Leinwand, Cooke, Noell, & Pollock, 2005).

4. *Hong Kong items were more likely to fall into the moderate or high cognitive complexity category (55%) compared with Massachusetts items (34%).* Performance on higher

cognitively complex items is an indicator of the ability to apply mathematical concepts to solving routine and nonroutine problems.

The following three Hong Kong assessment items illustrate features of items that make the Hong Kong assessment a more demanding assessment of students' deep mathematical understanding of Grade 3 mathematical concepts.

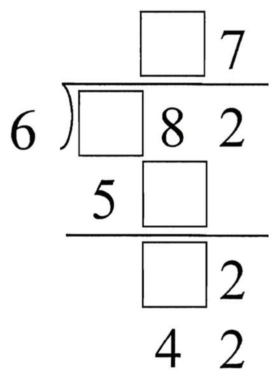
Order of Computational Problem:

340 – 500 + 460 = __________

This computation involves a difficult negative number problem if students carry out the subtraction sequentially (i.e., 340 – 500). However, if students understand instead to add out of sequence, 340 + 460, the resulting 800 produces an easy mental subtraction of 500 to yield an answer of 300.

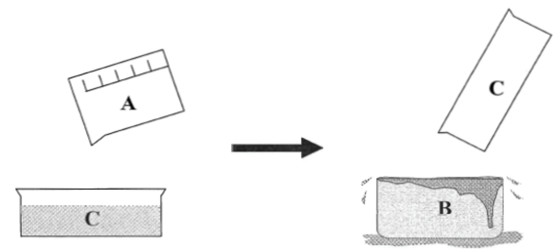
Missing Number Division Problem:

Fill in the boxes with the correct numbers.



This nonroutine multistep division problem with missing digits provides students with several appropriate solution paths to solve a long division problem by finding a solution that fits the partial answers presented in the item. For example, students could conclude that 9 is the missing digit in the quotient because 9 is the only number that will yield a beginning 5 in the first row under the dividend. Other students might start with the lowest box having to be a 4 since there is no remainder and then work backward to get the other numbers, thus providing an alternative approach. Working long division backward is a novel situation that requires a deep understanding of long division procedures.

Application of Measurement Principles Problem:



Container A is filled completely with water and all the water is poured into Container C. Then all the water in Container C is poured into Container B (the results are shown in the diagram above). Arrange Containers A, B and C in order, from the largest capacity to the smallest. Write the letters for the answer.

Answer: _________ , _________ , _________
(Largest) (Smallest)

This Hong Kong volume item involves the multistep application of the measurement principle of conservation in which the actual quantity (i.e., volume) of a material is invariant to the shape of the measuring instrument (i.e., container). In this situation, students must use Container A as the measurement unit and understand from the diagram that C is larger than A because Container C is not filled by the contents of Container A. Students also must understand that Container B is smaller than A because Container C holds A and when this volume of water is poured into B, Container B overflows. In addition, students must follow the physical order of steps indicated by the direction of the arrow (A, C, B), which is not the alphabetical order.

Overall, the Grade 3 mathematics topics covered by Hong Kong and Massachusetts assessments are, for the most part, similar. However, this international benchmarking comparison of assessments from the best in the United States against the best in the world yields new insights. It shows the greater depth of mathematical understanding required to solve many items on the Hong Kong assessment, compared with Massachusetts items. This expectation of deep understanding of mathematics concepts is a likely contributor to Hong Kong's achievement as the highest performer on TIMSS in the early grades.

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A goal of many state education systems is to achieve the now 30-year-old aim of *A Nation at Risk* to offer and attain a world-class mathematics education. One way to evaluate whether states are in fact offering a competitive, world-class mathematics education is to compare specific assessment questions, because these test items indicate the mathematics for which students and teachers are held accountable. This paper compares the spring 2007 assessment for Grade 3 in Massachusetts, the highest performing U.S. state on the National Assessment of Educational Progress (NAEP, 2007), with that for Hong Kong, the country with the highest mean performance on the 2007 Trends in Mathematics and Science Study (TIMSS) assessment, Grade 4 (Mullis et al., 2008).

The internal mathematics assessments of countries and states are unlike international assessments, such as TIMSS, that reflect mathematical content common across test-taking countries. A country's or a state's internal mathematics assessment indicates the mathematical content that is truly valued and demonstrates the degree of rigor and problem-solving capability actually expected of its students. In fact, the benchmarking to the education systems in high-performing countries has strong support in the joint publication of the U.S.'s National Governors Association, Council of Chief State School Officers, and Achieve, *Benchmarking for Success: Ensuring U.S. Students Receive a World-Class Education* (2008).

This paper examines assessments in the early elementary grades, where a strong foundation in basic mathematics concepts and procedures has to be developed, and on which more-advanced mathematics topics can build. It compares Hong Kong's Grade 3 assessment administered in June with that for Grade 3 in Massachusetts administered in May. The comparison of the content and characteristics of the Massachusetts assessment with those of the Hong Kong assessment suggests areas of difference that may guide Massachusetts and other states to reexamine their mathematics assessments. These differences between Hong Kong and Massachusetts assessments also are intended to stimulate each U.S. state to conduct its own comparison with Hong Kong, the world's Grade 4 best performer.

The source of the Hong Kong assessments is the unique international assessment database compiled for the Asian Pacific Economic Cooperation (APEC), an organization that represents 21 Pacific Rim countries. The Hong Kong internal assessments are available for Grades 3 and 9 on the APEC Wiki site (http://hrd.apecwiki.org). The focus of this study is on Hong Kong Grade 3. The Massachusetts items for Grade 3 can be downloaded from the Massachusetts Department of Elementary and Secondary Education Web site (http://www.doe.mass.edu/mcas/testitems.html).

The analysis examines all the Massachusetts and Hong Kong items for their distribution by content domain, item question types, real-world context, computational difficulty, and cognitive complexity.

The paper consists of seven sections. Section I describes the Massachusetts and Hong Kong mathematics assessments and the methodology employed to compare assessment items. Sections II through VI compare assessment items within each of five mathematical domains: numbers, measurement, geometry, algebra/patterns, and data and probability. Section VII presents a summary of findings and suggests areas to explore to strengthen the Massachusetts assessment relative to that of Hong Kong.

I. BACKGROUND AND METHODOLOGY

Comparing Hong Kong and Massachusetts Performance

As noted, this study selected Hong Kong because it has the highest mean performance on the 2007 TIMSS-4 international mathematics assessments and Massachusetts because it is the highest performing state on NAEP elementary (Grade 4) mathematics. But Massachusetts also participated as a country in the TIMSS-4 as part of the benchmarking component of the study, so its performance is directly comparable with that of Hong Kong. In fact, Massachusetts also performs relatively well internationally: ranking fourth among all participating countries or provinces, behind Singapore and Chinese Taipei, but ahead of the high-performing Japanese students and considerably ahead of the United States students' average (see Exhibit 1).

Hong Kong's major advantage in its TIMSS results over Massachusetts is that Hong Kong had a considerably higher percentage of its students achieving at the top levels of TIMSS performance benchmarks. Among Hong Kong students, 40% scored at the advanced level, compared with 22% of Massachusetts students. Similarly, 81% of Hong Kong students scored at or above the high level, compared with 63% of Massachusetts students. Benchmarking Massachusetts against the world's best Grade 4 performer provides insight into the types of items that Massachusetts could incorporate within its assessment and use to strengthen its instruction at the upper end of cognitive difficulty, to produce achievement more comparable with that of Hong Kong. Interestingly, only 10% of U.S. students overall achieved the advanced level and 40% achieved the high level or above, so the rest of the U.S. states would likely benefit even more by incorporating characteristics of both the Hong Kong and the Massachusetts assessments within their own assessments.

Exhibit 1. Ranking of Hong Kong and Massachusetts on 2007 TIMSS-4 Mathematics: Grade 4

Country/State	Rank	Average Scale Score	% at Advanced	% at High or Above	% at Intermediate or Above	% at Low or Above
Hong Kong	1	607	40	81	97	100
Singapore	2	599	41	74	92	98
Chinese Taipei	3	576	24	66	92	98
Massachusetts	4	572	22	63	92	99
Japan	5	568	23	61	89	98
United States	11	529	10	40	77	95

Source: Mullis et al., 2008.

Hong Kong and Massachusetts Assessments

If mathematics frameworks and standards identify what students *should* know and be able to do and represent the intended mathematics curriculum, the mathematics assessment items reflect what students are specifically *expected* to know and be able to do. In practical terms, the items on an assessment communicate what is actually valued and measured. What is measured tends to reflect what is taught. As such, assessments can exert a powerful influence over the actual content of instruction.

Care must be taken to judge assessments in terms of their purposes. For example, TIMSS attempts to assess the curriculum taught, whereas the Programme for International Students Assessment (PISA) international assessment of 15-year-olds is designed to evaluate students' ability to apply fundamental mathematics concepts within different real-world contexts irrespective of where students learned the necessary knowledge and skills. Thus, PISA would not be the best design for assessing students' understanding of more-advanced high school mathematics such as trigonometry. Similarly, the content of mathematics assessments will differ if they are intended to focus on the mathematical knowledge and skills that all students should have or if, alternatively, they are designed to assess the full range of students' mathematical abilities.

Also, the content of any particular assessment is influenced by whether it is high stakes and holds all students, teachers, or schools accountable for learning progress or performance or whether it is low stakes and serves primarily to inform students, schools, or school systems about performance as a way to focus improvement.

Hong Kong Basic Competency Assessment

The Hong Kong assessment is a basic competency assessment (BCA). It covers "the essential knowledge and skills that should be acquired by students in relation to the learning targets and objectives set out in the [mathematics] curriculum for each key stage" (Government of the Hong Kong Special Administrative Region, Education Bureau,n.d.). Hong Kong identifies three key stages. Key Stage 1 covers primary 1 to 3; Key Stage 2 covers primary 4 to 6; and Key Stage 3 covers lower secondary (1 to 3). Similar to the Curriculum Focal Points developed recently by the National Council of Teachers of Mathematics (2006), the Hong Kong assessment focuses on critical learning targets.

The BCA has two major purposes. One is to "enable teachers to understand students' learning needs and problems so as to facilitate timely assistance." The second purpose is to "provide the Government and school management with information on the performance of the students of a school in key learning areas (e.g., mathematics is one of the key learning areas) so that the Government will be able to provide support to those schools in need of assistance in order to enhance learning and teaching, and to monitor the effectiveness of education policies." (Government of the Hong Kong Special Administrative Region, Education Bureau,n.d.).

The BCA consists of two components, the student assessment and the system assessment (Exhibit 2). The student component is a formative assessment developed and administered by the classroom teacher from items in an electronic assessment item bank. Suitable items and tasks from the bank can be selected by teachers for assessing students' basic competencies. Items and tasks from the item bank are scored electronically, which reduces the workload of creating and scoring items. Instant reports that help teachers understand the performance of students in certain basic competencies also are available after the assessment. Teachers are then expected to take appropriate follow-up actions where performance weaknesses are identified.

Exhibit 2. The Two Components of the Hong Kong Basic Competency Assessment

Student Assessment	System Assessment
Judged by teachers on how to use	Coordinated and implemented by the central government
Conducted with computer	Conducted mostly on paper
Used to match with daily teaching	Conducted on specified date and location
Provides individual student information	Provides territory-wide information
Stores information within schools to be used by teachers as reference	Stores information centrally to be used as reference for policy making

Source: Government of the Hong Kong Special Administrative Region, Education Bureau, n.d.

The BCA system assessment is a low-stakes assessment designed to enhance student learning. The system assessment is a paper-and-pencil test that is administered near the end of Grade 3 and is hand scored, unlike the individual student computer-delivered assessment. The Primary 3 assessment has four equivalent forms and covers numbers, measurement, shape and space, and data handling (i.e., pictograms). Scores are reported for schools but not for individual students by name. Each school receives only its own scores, and schools are not to be compared in performance.

Massachusetts Comprehensive Assessment System

The Massachusetts Comprehensive Assessment System (MCAS):

- Tests all public school students in the spring of Grades 3 to 8 and Grade 10, including students who have disabilities and limited English proficient students;
- Measures performance on the basis of the Massachusetts Curriculum Framework learning standards;
- Reports on the performance of individual students, schools, and districts;

- Uses passing the Grade 10 tests in English language arts (ELA) and mathematics as one condition of eligibility for a high school diploma (in addition to fulfilling local requirements); and
- Holds schools and districts accountable, on a yearly basis, for meeting the annual progress they have made toward the objective of the No Child Left Behind law that all students be proficient in reading and mathematics by 2014.

The MCAS differs from the Hong Kong BCA in the following important respects:

- The MCAS is not designed as a basic competency assessment as is the BCA. Thus, even if the BCA is found more cognitively demanding, this should still be interpreted as a minimum estimate of Hong Kong's overall mathematics rigor.
- The MCAS is also used as a high-stakes assessment, although it too serves to inform instruction, whereas Hong Kong's assessment is only a low-stakes assessment designed to improve teaching and learning. As a result, the MCAS may include a greater percentage of less-demanding items to enable students of lower mathematics ability to pass the test.
- Individual student results on the MCAS are sent to schools, whereas no individual student results are reported for the BCA.
- Individual school performance results for the MCAS are published, although the Massachusetts Department of Elementary and Secondary Education warns against simple comparisons of schools serving different student populations.
- The MCAS is administered in Grades 3 to 8 and Grade 10, but equivalent testing in Hong Kong is administered only in Grades 3, 6, and 9.

These differences in testing purposes and context need to be kept in mind to understand the observed differences between the Hong Kong and Massachusetts assessments, but they do not invalidate the assessment comparisons. Indeed, the content of the Massachusetts high-stakes assessment may exert a stronger influence over mathematics instruction than does the Hong Kong assessment given the high- versus low-stakes nature of the MCAS relative to the BCA.

Methodology for Comparing Assessments

The current Hong Kong–Massachusetts comparison is for the Grade 3 mathematics assessments. All items on the Massachusetts assessment are included. Hong Kong administers four separate parallel forms covering the same mathematics topics, and we have randomly selected the items from one complete assessment.[2]

Each item was initially classified by mathematical strand. Hong Kong items cover the four strands of number, measurement, geometry (space and shape), and data handling. The MCAS items cover an additional strand of patterns, relations, and functions, which is the precursor to algebra. Each assessment item was further classified by major topic within a strand. For example, the numbers strand consists of these topics: ordering whole numbers, rounding whole numbers, place value, addition/subtraction, multiplication/division, fraction concepts, and fraction computation.

[2] Two items also were selected from one of the forms to fill gaps in the comparison with Massachusetts items, although the characteristics of these two items are not included in any of the analyses.

Each assessment item also was classified by question type—multiple choice, short closed constructed-response, and open constructed-response. Multiple-choice items, for which students select among responses, are easiest to score and are appropriate when there is no interest in having students show their work. Short closed constructed-response items are appropriate for assessing whether students can demonstrate or construct a solution to a problem, but the process or procedures are relatively straightforward and it is not necessary for students to show their work. Open constructed-response items are appropriate for assessing how well students understand mathematics solutions, explain their answers, and solve more complex problems. Multistep problems, some structured into steps and others unstructured, frequently employ open constructed-response items to assess students' understanding of how the students arrived at their solutions.

Items were further classified by several problem characteristics. The first classification is whether the problem is presented within a real-world context, a characteristic that has been associated with a challenging 21st century education. The second is computational difficulty, which is rated as low, medium, or high on the basis of such considerations as the number of digits in the operations, the need to regroup when computing, or the presence of medial zeros in subtraction and division, as well as on the basis of typical curricular expectations for U.S. third graders. Third, the item characteristic of greatest focus is cognitive complexity, a measure of depth of conceptual understanding and application of concepts to problem-solving. The cognitive complexity measure employs the three-part TIMSS categories of knowing, applying, and reasoning (Mullis, 2008):

- Level 1: *Knowing* "covers the facts, procedures, and concepts students need to know" through such mathematical behaviors as recall, recognize, compute, retrieve, and classify/order.

- Level 2: *Applying* requires knowledge and procedures to solve "routine problems" through the mathematical behaviors of selecting appropriate operations, representing mathematical information, generating a model, and implementing a set of mathematical instructions.
- Level 3: *Reasoning in nonroutine problem* situations involves the mathematical behaviors of analyze, generalize, synthesize/integrate, and justify.

All items for the two assessments and their classifications can be found in Appendix 2, and Exhibit 3 summarizes the key features of the two assessments.

Exhibit 3. Comparison of Key Features of Massachusetts and Hong Kong Grade 3 Mathematics Assessments

Characteristic	Massachusetts	Hong Kong
	MCAS Spring 2007 Released Items	2007 Territory-Wide Spring System Assessment
Time allocated	Not available	40 minutes
Total # of items	35	36*

Exhibit 3. Comparison of Key Features of Massachusetts and Hong Kong Grade 3 Mathematics Assessments (continued)

Characteristic	Massachusetts MCAS Spring 2007 Released Items	Hong Kong 2007 Territory-Wide Spring System Assessment
Items by strand**	Number: 17 (49%) Measurement: 4 (11%) Geometry: 4 (11%) Data: 6 (17%) Algebra: 4 (11%)	Number: 15 (42%) Measurement: 12 (33%) Geometry: 7 (19%) Data: 2 (6%) Algebra: 0 (0%)
Multiple-choice items	25 (71%)	5 (14%)
Constructed-response items	10 constructed response (all but two are short closed constructed-response)	31 constructed response (all but two are short closed constructed-response)
Items with graphics	20 (57%)	25 (69%)
Items within real-world contexts	19 (54%)	21 (58%)
Items by computational difficulty	Low: 34 (97%); Medium: 1 (3%); High: 0 (0%) Average (1, 2, 3) = 1.03	Low: 24 (63%); Medium: 13 (34%); High: 1 (3%) Average (1, 2, 3) = 1.39
Items by cognitive complexity***	Level 1: 23 (66%) Level 1+: 0 Level 2: 12 (34%) Level 2+: 0 Level 3: 0 Average: 1.34	Level 1: 12 (33%) Level 1+: 4 (11%) Level 2: 16 (44%) Level 2+: 3 (8%) Level 3: 1 (3%) Average: 1.68

* The typical form has 36 items; however, two addition matched items in the number strand were added as examples, but are not included in any of the summary exhibits, counts or percentages.

** This includes three Massachusetts number sentence "ordering" items as "number" despite their being classified by Massachusetts as "algebra."

*** Cognitive complexity: Level 1: knowing/recalling; Level 1+: knowing/recalling with some nuance; Level 2: applying; Level 2+: applying with some nuance; and Level 3: reasoning.

The following five sections compare the Hong Kong and Massachusetts assessments on these item characteristics applied to numbers, measurement, geometry, data and probability, and algebra. Each section begins with a discussion of context that describes the TIMSS findings, Hong Kong's BCA framework for that domain, and how it differs from the Massachusetts topic framework. (See Appendix 1 for the full Massachusetts Grade 3 framework.) The second part of each section highlights selected Hong Kong and Massachusetts assessment items on the item characteristics discussed above.

II. Numbers

Context

Hong Kong had the highest 2007 TIMSS-4 average score on the number strand of the Grade 4 TIMSS. Massachusetts had a respectable rank of fourth (Exhibit 4), with a score 35 points below that of Hong Kong. This is a statistically significantly difference.

Exhibit 4. Hong Kong and Massachusetts TIMSS Scores on the Number Domain: Average Score and Rank

	Average Score for the Domain*	Rank
Hong Kong	606	1
Massachusetts	571	4

* TIMSS average score is set at 500 and a standard deviation of 100. The Hong Kong–Massachusetts difference is about one third of a standard deviation.

Source: TIMSS 2007.

An important initial step in exploring the underlying differences in the assessment items is to compare the content of the mathematics frameworks on which the assessments are based. Exhibit 5 displays the Hong Kong topics for the Grade 3 numbers domain. The standard introductory mathematics topics are addressed: understanding place value and ordering whole numbers up to five digits; understanding and using the four arithmetic operations along with mixed operations with three digits as the maximum number size; and understanding and comparing fractions with the same numerators or denominators.

The topics in the Massachusetts Grade 3 framework (see Appendix 1) are similar to those in Hong Kong's BCA framework, with two notable exceptions. First, the Massachusetts framework limits Grade 3 students to a maximum of four-digit numbers (i.e., 9999), compared with the larger five-digit numbers required by the Hong Kong framework at Grade 3. Second, the Massachusetts framework gives explicit attention to rounding and estimation, but Hong Kong's BCA framework does not. Nonetheless, rounding is a topic on the Hong Kong assessment, although estimation is not.

Exhibit 5. Hong Kong Basic Competency Assessment: Grade 3 Numbers

Five-Digit Whole Numbers
• Recognize the place values: units, tens, hundreds, thousands, and ten thousands. • Read, write, and order numbers up to five digits.
Arithmetic Operations
• Perform addition (with numbers up to three digits, not involving carrying in three steps but involving the commutative and associative properties of addition). • Perform subtraction (with numbers up to three digits). • Perform multiplication (with numbers up to one digit by three digits, involving the commutative property of multiplication). • Perform division (with one-digit divisor and three-digit dividend). • Perform mixed operations of: (a) addition and subtraction (with numbers up to three digits, involving small brackets); (b) multiplication (numbers less than 10) and addition (numbers less than 10) and subtraction.

Exhibit 5. Hong Kong Basic Competency Assessment: Grade 3 Numbers (continued)

Arithmetic Operations (continued)
• Solve problems involving mixed operations. • Solve problems involving addition, subtraction, multiplication, and division in the calculation of money (not involving mixed operations).
Fractions
• Understand the concept of fractions as a part of one whole. • Recognize the relationship between fractions and the whole. • Compare fractions with same denominators or same numerators.

Source: Hong Kong BCA descriptors, retrieved January 2009 from http://cd1.edb.hkedcity.net/cd/eap_web/bca/index3.htm.

Findings

The numbers strand of the Hong Kong and Massachusetts assessments are quantitatively compared on four dimensions: the number and percentage of number items in the total assessment; item type (e.g., percentage of constructed-response items); the average difficulty of computational problems; and the average cognitive complexity of the questions. The findings are summarized in Exhibit 6.

Exhibit 6. Summary of the Number Strand

Dimension	Massachusetts	Hong Kong
Total number of items	17	15
Percentage of total items	49%	42%
Item type	Twelve items are multiple choice and five are short closed constructed-response	Two items are multiple choice and 13 are constructed-response (11 short and 2 open)
Computational difficulty	Sixteen items have low computational difficulty and one has medium difficulty	Two items have low computational difficulty and 13 have medium difficulty
Cognitive complexity	Twelve items are Level 1 and five are Level 2	Three items are Level 1, one is Level 1+, nine are Level 2, and two are Level 2+

- *Percentage of number items.* Both the Hong Kong and the Massachusetts assessments stress items in the numbers domain, reflecting its importance as a foundation for early mathematics learning. The percentage of total test items in this domain is 42% for Hong Kong and 49% for Massachusetts. However, Massachusetts classifies four of these items that involve number sentences under patterns, relations, and algebra. Thus, using Massachusetts' coding, the percentage of items in the number domain drops to 37%.
- *Item type.* Hong Kong number items are considerably more likely to require students to provide short or long constructed responses than multiple-choice responses: 13 out of 15 (87%) for Hong Kong, compared with only 5 out of 17 (29%) for Massachusetts.

- *Computational difficulty.* Thirteen out of 15 (87%) Hong Kong items were of moderate computational difficulty, often because Hong Kong items may involve medial zeros. Only one Massachusetts item (6%) was considered beyond the low computational requirement.
- *Cognitive complexity.* The percentage of items with middle or high cognitive demand was 73% on the Hong Kong assessment, compared with 29% on the Massachusetts assessment.

The following examples illustrate the greater mathematical demand of the Hong Kong number items, compared with similar items on the Massachusetts assessment.

Place value—Hong Kong items involve multiple conditions (Exhibit 7)

The idea that the value of a digit depends on its place within a number can be perplexing to young students, but it lies at the core of number understanding. A Hong Kong place-value item involves five digits and requires ordering digits under *multiple conditions*. That is, as shown in Exhibit 7, instead of ordering only five numbers to form the smallest number, which would end with the largest numeral 8, Hong Kong requires finding the smallest *odd* number, which must end with the largest odd numeral 5. The Massachusetts place-value item is simpler in that it includes no zeros and does not add a condition about the number being odd as well as smallest.

Exhibit 7. Number: Place Value

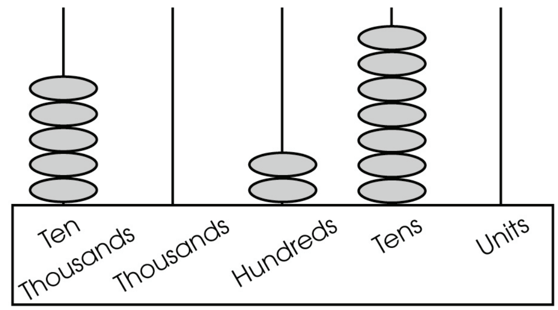
Massachusetts	Hong Kong
Use the number tiles from your tool kit to answer the following question. Alan has the number tiles shown below. **4 | 7 | 8 | 1** a. Use all of Alan's number tiles to make the four-digit number with the **smallest** value. Use each number tile only one time. Write the number in the boxes below. [ ] [ ] [ ] [ ] b. What is the value of the digit 7 in the number you made? Explain your answer.	**3 | 0 | 8 | 2 | 5** Arrange the 5 number cards above to form the smallest 5-digit **odd** number. Answer: [ ] [ ] [ ] [ ] [ ] **NOTE: This item is from an alternate form of the test.**

Ordering whole numbers—Hong Kong items involve different representations (Exhibit 8)

The ordering of whole numbers is basic to understanding numbers and applying place-value concepts. As shown in Exhibit 8, a Hong Kong item involves a complicated comparison of numbers in three *different representations* (pictures, words, symbols) that must be written as numerals in order. The Hong Kong item also involves five digits and is short answer, not multiple choice. In contrast, a Massachusetts item presents a table displaying books read in three classes and requires the student only to select the number between two given four-digit numbers.

The “books read” context adds reading load, but the table presents the information clearly and accessibly.

Exhibit 8. Number—Ordering Whole Numbers

<table>
<tr><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr>
<td>
The table below shows how many books three classes read.
<table>
<tr><th colspan="2">Books Read</th></tr>
<tr><th>Class</th><th>Number of Books</th></tr>
<tr><td>Ms. Potter</td><td>1023</td></tr>
<tr><td>Ms. Hogan</td><td>?</td></tr>
<tr><td>Mr. Garcia</td><td>1067</td></tr>
</table>
Ms. Hogan’s class read more books than Ms. Potter’s class and fewer books than Mr. Garcia’s class.
Which of these could be the number of books Ms. Hogan’s class read?
a. 1074 b. 1166 c. 1005 d. 1062
</td>
<td>
Arrange the following numbers from the largest to the smallest (give the answers in numerals).


b. Fifty-two thousand and seven
c. 50720
______________, ___________, ____________
(Largest) (Smallest)
</td>
</tr>
</table>

Subtraction—Hong Kong items involve multiple solution strategies (Exhibit 9)

Students need to understand that there is often more than one way to solve a mathematical problem and that some solutions may be more computationally efficient or advantageous to carry out than other solutions. The two subtraction problems in Exhibit 9 differ in their demands to assess different solution strategies. The Hong Kong item assesses an understanding of the different ways to carry out subtraction. If Hong Kong students carry out the arithmetic in sequence, they encounter 340 – 500, a difficult negative number problem. However, if they apply properties of addition and subtraction, they understand that they can first add 340 and 460 to obtain 800, leaving a simple mental subtraction of 800 – 500. The Massachusetts item simply requires students to identify two numbers from a table (30 and 8) and recognize that the context asks for a difference (how many more?) that emerges from a straightforward 30 – 8.

Exhibit 9. Number: Subtraction

<table>
<tr><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>Mr. Wilson's class made the chart below to show the number of birds that ate at a bird feeder on five days.

Birds That Ate at the Bird Feeder
<table>
<tr><th>Day</th><th>Number of Birds</th></tr>
<tr><td>Monday</td><td>8</td></tr>
<tr><td>Tuesday</td><td>18</td></tr>
<tr><td>Wednesday</td><td>30</td></tr>
<tr><td>Thursday</td><td>12</td></tr>
<tr><td>Friday</td><td>20</td></tr>
</table>

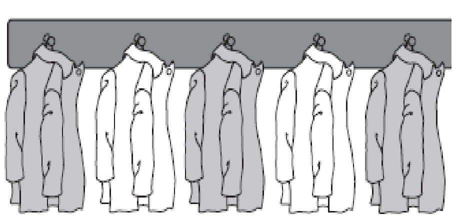
How many more birds ate at the bird feeder on Wednesday than on Monday?</td><td>340 – 500 + 460 = __________</td></tr>
</table>

Fractions—Hong Kong items involve understanding concepts from multiple perspectives (Exhibit 10)

As shown in Exhibit 10, a Hong Kong item assesses the understanding of fractions from multiple perspectives. Specifically, students face four different pairs of fractions and must identify the correct order relationship between the two members within each pair. Together, the four items assess students' understanding of fractions with the same denominators, the same numerators, and equal numerators and denominators. The items also require students to apply their understanding of "less than," "equal to," or "greater than." By contrast, the Massachusetts item requires only that students understand the basic representation of the fraction as part of a set.

Exhibit 10. Number: Fractions

<table>
<tr><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>The coats shown below are hanging on coat hooks.



What fraction of the coats are white?
Write your answer in the Answer Box below.

[ ]</td><td>Fill in the boxes with " > ", " < " or " = ".

(a) $1 \ \square \ \frac{10}{10}$

(b) $\frac{1}{5} \ \square \ \frac{1}{8}$

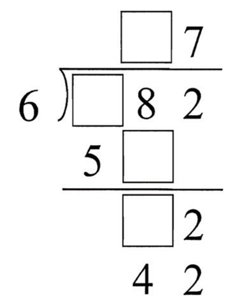
(c) $\frac{3}{11} \ \square \ \frac{9}{11}$

(d) $\frac{2}{2} \ \square \ 2$</td></tr>
</table>

Numerical reasoning—Hong Kong items involve nonroutine situations (Exhibit 11)

Deep mathematical understanding emerges when students are able to use their understanding of mathematical concepts, procedures, and strategies to solve a problem under unfamiliar conditions. As shown in Exhibit 11, the closest that a Massachusetts Grade 3 number item comes to being "nonroutine" is a "guess-my-number" situation requiring the identification of the one number that satisfies three number property constraints. In contrast, the Hong Kong item presents students with a partially completed long division problem with missing numbers in the dividend, intermediate calculations, and quotient, thereby requiring the student to fill in the blanks by using a mature understanding of the products and differences in the procedure. Students who are familiar only with mechanical long division will have serious difficulty in filling in the missing numbers.

Exhibit 11. Number: Numerical Reasoning

Massachusetts	Hong Kong
Maria is thinking of a number. The clues for her number are shown below: • It is a multiple of 5. • It is an even number • It is less than 18. Which of these could be Maria's number? a. 5 b. 20 c. 8 d. 10	Fill in the boxes with the correct numbers.  **NOTE: This item is from an alternate form of the test.**

Multiplication—Hong Kong items involve mixed operations (Exhibit 12)

Another important understanding is the distinction between and among operations. A Hong Kong item, shown in Exhibit 12, involves more-complicated arithmetic expressions and combines multiplication with addition instead of using only one operation. It also requires understanding that the traditional ordering of operations without parentheses requires that the multiplication operation be carried out first. In this case, the sequential operation from left to right would yield an incorrect answer. In contrast, the Massachusetts item poses a straightforward number sentence problem that requires knowing only the multiplication fact (8 × 3 = 24).

Exhibit 12. Number: Multiplication

Massachusetts	Hong Kong
Cindy wrote the number sentence below. $\boxed{?} \times 3 = 24$ In the Answer Box below, write the missing number that makes Cindy's number sentence true. $\boxed{}$	$188 + 78 \times 4 =$ ________

Estimation—Massachusetts items involve a straightforward rounding to make estimates (Exhibit 13)

Estimation is the one major Grade 3 assessment topic that is covered in the Massachusetts assessment but not in the Hong Kong assessment. This finding is consistent with a general emphasis in the United States on real-world mathematical examples, of which estimation is a common example. However, the Massachusetts estimation item in Exhibit 13 is a straightforward problem in which students need to round up the cost of each shirt from $8.95 to $9.00 and multiply by 4.

Exhibit 13. Number: Estimation

Massachusetts	Hong Kong
Brianna bought 4 shirts. Each shirt cost $8.95. Which estimate is closest to the total cost of the shirts that Brianna bought? a. $32 b. $36 c. $38 d. $40	**The Hong Kong test includes no estimation items.**

Summary

The Massachusetts assessment covers the essential number topics and includes many items that require reading and interpretation, but overall its items are not as mathematically demanding as those the Hong Kong students face. The Hong Kong assessment involves larger numbers and more-complicated arithmetic when compared with that of Massachusetts. The cognitive complexity is higher in Hong Kong number strand items, and the items are more likely to involve multiple steps, multiple representations, examining concepts from different perspectives, and occasionally handling novel situations.

III. Measurement

Context

The definition of measurement is the "estimation of the magnitude of some attribute of an object, such as its length or weight, relative to a unit of measurement" (en.wikipedia.org/wiki/Measurement). Measurement involves (a) determining the attribute to be measured, (b) selecting an appropriate unit of measurement, (c) applying or counting the number of units typically with some measurement instrument, and (d) interpreting the results.

TIMSS combines the scores for measurement with geometry in reporting country scores by content area (Exhibit 14); however, the number strand of TIMSS also includes some measurement items. The Hong Kong measurement/geometry combined score is the highest among all Grade 4 test-takers, with its performance in the measurement/geometry area about the same as the Hong Kong overall score. The Massachusetts score of 564 places it fourth among TIMSS Grade 4 test-takers and about one third of a standard deviation below that of Hong Kong. Massachusetts' performance in measurement and geometry does not differ from Massachusetts' overall performance.

Exhibit 14. Hong Kong and Massachusetts TIMSS Scores on the Measurement and Geometry Domains: Average Score and Rank

	Average Score for the Domain*	Rank
Hong Kong	599	1
Massachusetts	564	4

* TIMSS average score is set at 500 and standard deviation of 100. The Hong Kong–Massachusetts difference is about one third of a standard deviation, an amount that is typically considered educationally significant.
Source: TIMSS 2007.

Hong Kong's BCA identifies five measurement objectives that students should learn by the end of Primary 3. The Hong Kong objectives are organized around different measurement units covering money, length and distance, time, weight, and capacity (Exhibit 15).

Exhibit 15. Hong Kong Basic Competency Assessment: Grade 3 Measurement Domain

Money
• Identify Hong Kong money, read price tags, and exchange money
Length and Distance
• Compare the length and distance between objects directly, using improvised units (e.g., paper clip, arm length); using millimeter, centimeters, and meters, and kilometer
Time
• Tell dates and days of week; tell time from digital and analog clock face; record duration in hours, minutes, and seconds; apply 24-hour time
Weight
• Compare weights directly, measure and compare weights using improvised units, grams, and kilograms using appropriate tools and units
Capacity
• Compare the capacity of containers directly, using improvised units, liter and millimeter, and appropriate tools

The Massachusetts measurement standards cover the same topics as those of Hong Kong, with two exceptions. First, Massachusetts includes conversions within the U.S. customary system or the metric system, but not between systems. Hong Kong uses metric units, so it does not cover the U.S. customary system of measurement units or conversions within the this system. Second, the Massachusetts standards explicitly mention estimation but the Hong Kong standards do not.

Findings

The findings, with respect to the proportion of measurement items, item type, computational difficulty, and cognitive complexity, are summarized in Exhibit 16.

Exhibit 16. Summary of the Measurement Strand

Dimension	Massachusetts	Hong Kong
Total number of items	4	12
Percentage of total items	11%	33%
Item type	Three items are multiple choice and one is short closed constructed-response	Two items are multiple choice and 10 are short closed constructed-response
Computational difficulty	All four items have low computational difficulty	Eleven items have low computational difficulty and one has medium difficulty
Cognitive complexity	Three items are Level 1 and one is Level 2	Two items are Level 1, three are Level 1+, six are Level 2, and one is Level 3

- *Proportion of measurement items.* The Hong Kong assessment disproportionately stresses measurement and Massachusetts does not: 33% for Hong Kong items, compared with 11% for Massachusetts.

- *Item type.* As with numbers, Hong Kong number items are more likely to require students to provide a short answer rather than select a response: 83% of Hong Kong items, compared with 25 % for Massachusetts items.

- *Computational difficulty.* Unlike items in the numbers domain, measurement items involve only very simple computations. Only one of 12 (8%) of Hong Kong items and none of the Massachusetts items were more than low computational difficulty.

- *Cognitive complexity.* The percentage of measurement problems with middle or high cognitive demand was 58% on the Hong Kong assessment, compared with 25% on the Massachusetts assessment.

In addition, despite the similarity between the Hong Kong and Massachusetts Grade 3 frameworks in measurement, entire topics are addressed on the Hong Kong assessment that are not addressed at all on the Massachusetts assessment. These topics include length, measurement concepts, measurement conversions, calendar, angle size, and map/directions.

The following examples illustrate the typically greater cognitive complexity of the Hong Kong items, compared with Massachusetts items in different aspects of the measurement strand.

Measurement unit—Hong Kong items involve generating the appropriate unit of measurement (Exhibit 17)

Determining the appropriate unit of measurement for a particular attribute is an essential part of any measurement process. The Massachusetts item shown in Exhibit 17 is a very basic one that requires students to select the unit appropriate to measure height from among four multiple-choice responses where the distracters relate to volume or weight. The Hong Kong item is a far more demanding constructed-response task that requires students to generate the appropriate weight unit for different items given a specific number of units.

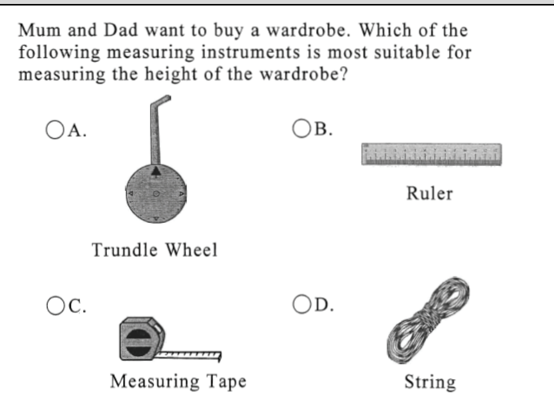
Exhibit 17. Measurement—Units

Massachusetts	Hong Kong
Which unit can Sara use to measure the height of the snow in her backyard? a. gallon b. pound c. inch d. ounce	Fill in each of the following blanks with a suitable unit for weight. a. A television set weighs about 20 __________. b. A piece of cake weighs about 150 _________. c. A bag of barbecue charcoal weighs about 5 _______.

Measurement instrument—Hong Kong items involve selecting the most appropriate tool among similar instruments (Exhibit 18)

Measurement is carried out using tools. Some are simple instruments such as a ruler, a balance scale, or speedometer. The Hong Kong Grade 3 assessment contains several items requiring students to select the appropriate measurement instrument or demonstrate an understanding of its use. One Hong Kong item, shown in Exhibit 18, challenges students to pick the most appropriate instrument to measure the height of a wardrobe among four instruments, each of which can measure length. The Massachusetts assessment contains no items involving measurement tools.

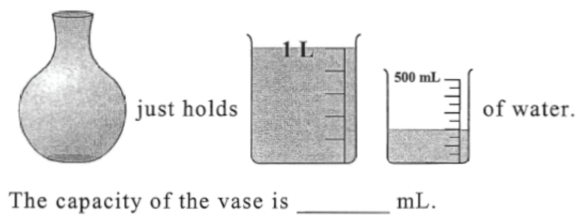
Exhibit 18. Measurement—Instruments

Massachusetts	Hong Kong
The Massachusetts test includes no measuring instrument items.	

Conversion of units—Hong Kong items involve applying conservation of quantity under different units (Exhibit 19)

A principle of measurement that students need to understand is the principle of conservation. That is, when different size units measure the same attribute (e.g., volume), the units can be converted without changing the actual quantity being measured. Massachusetts has no items involving converting units, but Hong Kong has an interesting constructed-response item, shown in Exhibit 19, that shows a vase of unknown capacity whose liquid contents fill a 1-liter container and a portion of a 500-milliliter container. The problem is also challenging in that students must read and combine liter and milliliter scales.

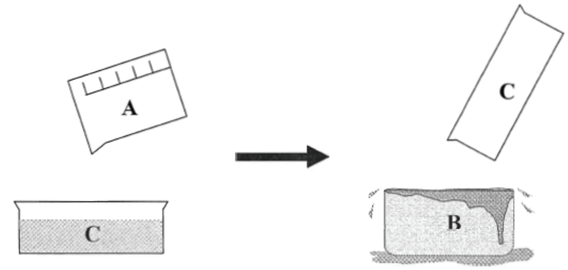
Exhibit 19. Problems in Measurement—Conversion

Massachusetts	Hong Kong
The Massachusetts test includes no conversion items.	

Transitivity—Hong Kong items involve nonroutine problems (Exhibit 20)

Transitivity means that when objects are measured using the same unit, then A is related to C as the combination of A is related to B and B is related to C. For example, if Desk A is wider than Desk B and Desk B is wider than the doorway, then Desk A is wider than the doorway. Massachusetts has no items about transitivity of measurement, but Hong Kong presents a nonroutine problem, shown in Exhibit 20, that involves two comparisons of the volume of water contained by three different containers. Students have to carefully read the picture to understand that Container C holds more water than a full Container A and that when the water in Container C is poured into Container B, it overflows. Students then need to apply transitivity logic to solve the problem.

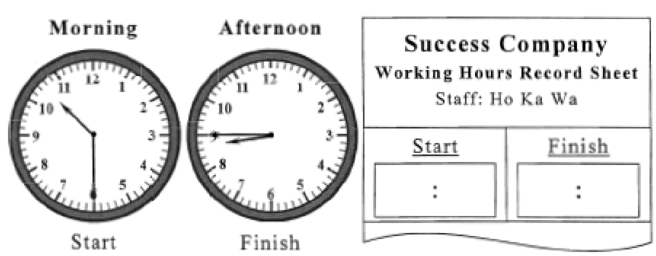
Exhibit 20. Problems in measurement—Transitivity

Massachusetts	Hong Kong
The Massachusetts test includes no transitivity items.	 Container A is filled completely with water and all the water is poured into Container C. Then all the water in Container C is poured into Container B (the results are shown in the diagram above). Arrange Containers A, B and C in order, from the largest capacity to the smallest. Write the letters for the answer. Answer: _________ , _________ , _________ (Largest) (Smallest)

Elapsed time and calendar date—Hong Kong and Massachusetts items involve multiple steps (Exhibits 21 and 22)

Time problems involving clocks may require students to correctly read an analog or a digital clock or to complete the more-complicated application to calculate elapsed time differences. Massachusetts and Hong Kong assessments both include higher-level problems for which students need to compute elapsed time. The Massachusetts item shown in Exhibit 21 involves a complicated two-step process of moving the clock forward 30 minutes and then back 10 minutes. The Hong Kong item involves transforming an analog time to digital in the morning and translating afternoon analog time to digital 24-hour time.

Exhibit 21. Measurement—Elapsed Time

Massachusetts	Hong Kong
The clock below shows the time that Mr. Stone put a cake in the oven.  a. The cake needs to bake for 30 minutes. At what time will the cake be done? b. Mr. Stone also wants to bake rolls. The rolls only need to bake for 10 minutes. At what time should Mr. Stone put the rolls in the oven so that they will be done at exactly the same time as the cake?	The clocks below show the time when Mr. Ho started and finished work on a certain day. Fill in the Working Hours Record Sheet for Mr. Ho using the "24-hour time." 

The Hong Kong assessment also includes a two-part, constructed-response calendar item, shown in Exhibit 22, whereas the Massachusetts assessment has no calendar items. The first part involves a simple calendar reading that requires knowing the date of the next Wednesday. However, the second part calls for a more-complicated counting of days ahead on a calendar that requires moving from December 2006 to a new year and month (January 2007) not shown on the calendar.

Exhibit 22. Measurement—Calendar

<table>
<tr><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>The Massachusetts test includes no calendar items.</td><td>
<table>
<tr><th colspan="7">December 2006</th></tr>
<tr><td>Sun</td><td>Mon</td><td>Tue</td><td>Wed</td><td>Thu</td><td>Fri</td><td>Sat</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>1</td><td>2</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td></tr>
<tr><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td></tr>
<tr><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr>
<tr><td>31</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
a. Mum bought a washing machine on the 14th of December and arranged to have it delivered on the Wednesday, which was the
________ of __________.
(month)

b. There were 11 days of holidays for Christmas and New Year. The holidays began on the 23rd of December. After the holidays, the pupils went back to school again on the
________ of __________, _______.
(month) (year)
</td></tr>
</table>

Money and making change—Hong Kong items involve multiple representations (Exhibit 23)

The Massachusetts assessment has a simple recall item, shown in Exhibit 23, for which students must convert quarters and dimes to pennies. In contrast, Hong Kong's multistep money problem involves translating a word representation of a price tag into numerals and then calculating and representing the change received using different paper and coin representations of money.

Exhibit 23. Measurement—Money

Massachusetts

The table below shows how many coins each child in the Jones family has.

Children's Coins

Child	Coins
Byron	3 quarters
Pam	100 pennies
Linda	4 quarters
William	50 pennies

Which two children have the same amount of money?

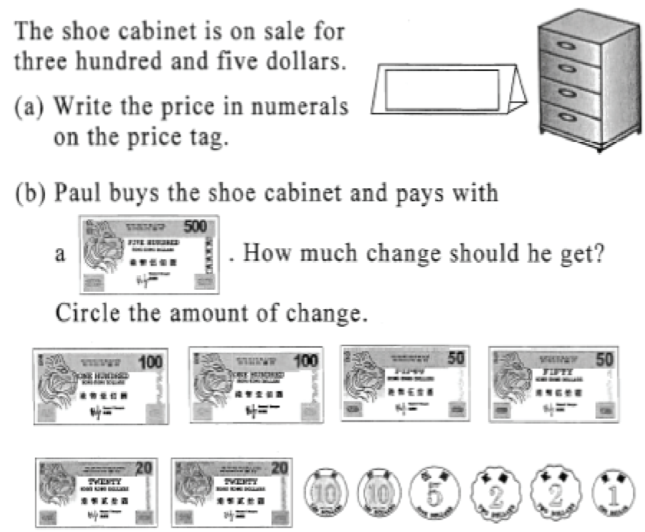
a. Linda and William

b. Pam and William

c. Byron and Pam

d. Pam and Linda

Hong Kong



Summary

Overall, Hong Kong's BCA disproportionately stresses measurement items in terms of the proportion of problems. The Massachusetts assessment does not. Hong Kong's measurement items are more likely to test the application of concepts, whereas the Massachusetts items mainly test recall and the simple application of definitions and concepts. Thus, the Hong Kong assessment requires students to select appropriate units or instruments, apply measurement principles such as transitivity, determine dates that cut across years, and make appropriate change involving different money representations. These measurement problems require students to apply, adapt, or integrate knowledge of measurement concepts, tools, and principles often in multistep problems that assess key conceptual understanding within the measurement strand.

IV. Geometry

Context

Geometry is one of the oldest formal topics in mathematics. The Greek mathematician Euclid wrote his 13-volume *Elements* about 300 B.C. Geometry at Grade 3 typically includes introductory material, in which students are learning basic definitions and properties of shapes. As noted, TIMSS combines measurement and geometry reporting of scores because of insufficient items to break out the two content areas separately.

Hong Kong's BCA framework (Exhibit 24) for Grade 3 geometry covers two-dimensional (2-D) and three-dimensional (3-D) shapes, lines, and angles. Hong Kong's framework also includes the four directions of a compass—a conceptual introduction to coordinate geometry. The cognitive demands in the BCA framework are at a very introductory level. For example, students are required to identify shapes or to recognize and compare angles, but they are not required to do much in the way of applying their geometry knowledge.

Exhibit 24. Hong Kong Basic Competency Assessment Framework: Grade 3 Geometry

2-D Shapes
• Identify 2-D shapes intuitively: triangles, quadrilaterals, trapeziums, parallelograms, pentagons, hexagons, squares, rectangles, rhombuses, and circles • Recognize the simple characteristics of triangles (e.g., three sides, three angles), including right-angled triangles, isosceles triangles, and equilateral triangles • Group 2-D shapes • Describe the relative positions of two 2-D shapes using "left," "right," "above," and "under"
3-D Shapes
• Identify prisms, pyramids, and spheres • Group 3-D shapes • Compare objects according to their lengths, widths, heights, and thicknesses • Describe the relative positions of two 3-D shapes using "front," "behind," "left," "right," "above," and "under"
Lines
• Identify straight lines, curves, parallel lines, and perpendicular lines
Angles
• Recognize angles and right angles • Compare sizes of angles
The Four Directions
• Recognize the four directions—east, south, west and north—with the use of a compass

The Massachusetts Grade 3 geometry framework mirrors the Hong Kong framework in the inclusion of 2-D and 3-D shapes, lines, and angles. Massachusetts does not cover the four directions, but it expands on Hong Kong's geometry Grade 3 requirements by including two additional topics:

- Using ordered pairs of whole numbers and/or letters, locate and identify points on a grid, an important skill in specifying locations.
- Predict and explain the results of taking apart and combining two-dimensional shapes.

Findings

The findings, with respect to the proportion of geometry items by item type, computational difficulty, and cognitive complexity, are summarized in Exhibit 25.

- *Proportion of geometry items.* The Hong Kong assessment contains 19% geometry problems and the Massachusetts assessment only 11%.
- *Item type.* The Hong Kong geometry items continue to emphasize constructed response, with six of seven (86%) of Hong Kong's problems requiring students to construct an

answer. Only 50% of Massachusetts items were constructed response, continuing the emphasis on multiple-choice format.

- *Computational difficulty.* Arithmetic is not required for any of the items in this strand.
- *Cognitive complexity.* The percentage of problems with middle or high cognitive demand was only 14% on Hong Kong's assessment, compared with 50% on the Massachusetts assessment.

Exhibit 25. Summary of the Geometry Strand

Dimension	Massachusetts	Hong Kong
Total number of items	4	7
Percentage of total items	11%	19%
Item type	Two items are multiple choice and two are short closed constructed-response	One item is multiple choice and six are short closed constructed-response
Computational difficulty	No computation is required for any of these items	No computation is required for any of these items
Cognitive complexity	Two items are Level 1 and two are Level 2	Six items are Level 1 and one is Level 2

In addition, the Hong Kong assessment includes an item on symmetry, a topic not addressed on the Massachusetts assessment.

Unlike the number and measurement items, most Hong Kong geometry items are at the low end (Level 1) of cognitive complexity, and the Massachusetts problems are more cognitively challenging.

The following discussion illustrates the items from the geometry strand at the basic and more-advanced cognitive levels.

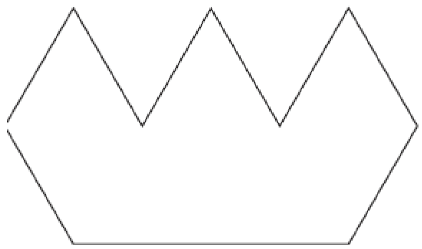
2-D shapes—Massachusetts items involve arranging elements to build a composite figure and Hong Kong items require constructing a figure that meets certain conditions (Exhibit 26)

These geometry items, shown in Exhibit 26, are both of higher cognitive complexity and require students to construct a figure from smaller shapes or construct a figure meeting certain conditions. The Massachusetts item requires students to identify the shape pieces needed to construct a given irregular shape. This multistep problem has students first identify different ways to break up the larger figure into smaller shapes. Then students need to look at the multiple-choice solution possibilities and determine how these components combine to make up the larger figure. The students do have the shapes to actually manipulate; however, the problem is made more difficult in that the same piece is shown in different orientations, and students have to select the proper rotation or, in one case, mentally rotate a piece because none is in the right position. The Hong Kong item also requires a construction, in this case, the selection of three points on a grid needed to form a right triangle.

Exhibit 26. Geometry Problems—2-D Shapes

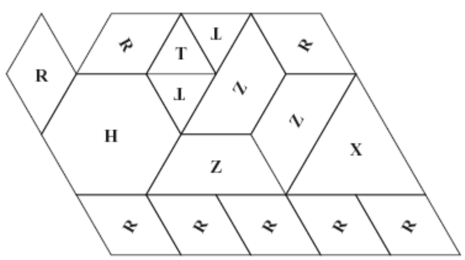
<table>
<tr><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr>
<td>
Use shapes R, T and Z from your toolkit to answer this question.

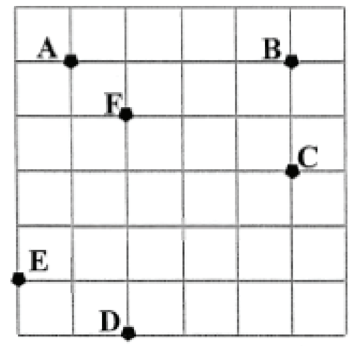
Which set of four shapes could you use to make the figure below?



a. R, R, T, Z
b. R, R, R, Z
c. R, R, Z, Z
d. R, R, T, T

Toolkit shapes



</td>
<td>


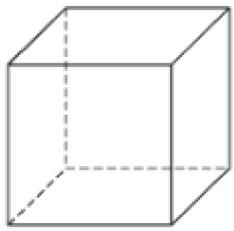
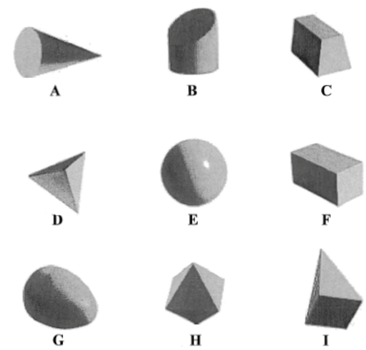
Martin wants to make a right-angled triangle on the pin-board using an elastic band. Which points on the pin-board should he choose? Write the letters for the answer.

Answer: ________________________
</td>
</tr>
</table>

3-D shapes—Hong Kong items involve multiple representations and extraneous information (Exhibit 27)

Both 3-D shape items shown in Exhibit 27 involve recall of shape definitions. The Massachusetts item poses a straightforward problem about the number of corners on a cube. Although the Hong Kong item is in one sense a straightforward problem about identifying pyramids, prisms, cones, and spheres, the choice among the shapes requires finding those features that are critical for differentiating among 3-D shapes. Hong Kong also includes a truncated cylinder (e.g., Figure B) and an ellipsoid (e.g., Figure G) that are not "named" and should not be selected.

Exhibit 27. Geometry Problems—3-D Shapes

Massachusetts	Hong Kong
A cube is shown below.  How many corners does a cube have? Write your answer in the answer box below.	Study the following 3-D shapes. Write the letter(s) for the answer.  a. Pyramid(s): ____________ b. Cone(s): ____________ c. Prism(s): ____________ d. Sphere(s): ____________

Summary

The introductory nature of geometry at Grade 3 is reflected in the assessment items. Yet, even at the introductory level, both Massachusetts and Hong Kong assessments include challenging geometric problems that involve the construction and manipulation of figures that require students to apply what they know, not simply recall facts.

V. Data and Probability

Context

Statistics is the mathematics domain "pertaining to the collection, analysis, interpretation or explanation, and presentation of data" ((en.wikipedia.org/wiki/Statistics).. Statistics provides a natural connection between the world of mathematics and the outside world of events and data that students encounter in their daily lives. Statistics is also a way for students to understand and judge the quality of information and to recognize misinformation. Because statistics also involves uncertainty, it is related to probability, a topic frequently taught as part of the statistics strand.

Hong Kong has a positive advantage over Massachusetts in the TIMSS category "data display" (Exhibit 28), but the advantage is comparatively small: 14 points, or about 40% of the advantage for Hong Kong on the numbers and measurement/geometry domains. For "data display" the Hong Kong performance is below Hong Kong's overall TIMSS average, whereas that of Massachusetts is about the same as its TIMSS average.

Exhibit 28. Hong Kong and Massachusetts TIMSS Scores on "Data Display": Average Score

	Average Score for the Domain*	Rank
Hong Kong	585	1
Massachusetts	571	4

* TIMSS average score is set at 500 and standard deviation of 100. The Hong Kong–Massachusetts difference is about one seventh of a standard deviation, an amount that is considered typically considered educationally significant.

Source: TIMSS 2007.

The Hong Kong mathematics framework for data and probability is quite limited (Exhibit 29). The Hong Kong framework calls the area "data" and does not cover probability. The only topics under data are reading and constructing pictograms, which are charts that use pictures to represent number quantities.

Exhibit 29. Hong Kong Basic Competency Assessment Framework: Grade 3 Data

• Read and interpret simple pictograms with a one-to-one representation.
• Construct pictograms using a one-to-one representation.

The Massachusetts framework defines the content area as "data analysis, statistics, and probability," indicating a far more expansive conception of this domain at Grade 3 than Hong Kong employs. This is illustrated in the following Massachusetts topics that are not covered by the Hong Kong assessments:

- Match representations of a data set in the forms of tables, line plots, pictographs, tallies, or bar graphs with the actual data set.
- Construct and draw conclusions from representations of data sets in the forms of tables, line plots, pictographs, tallies, and bar graphs.
- List and count the number of possible combinations of objects from two sets; for example, how many different outfits can one make from a set of two sweaters and a set of three skirts?

Thus, Massachusetts, in addition to covering pictograms, expects students to be familiar with tables, line plots, and bar graphs and to draw conclusions from representations of each. In addition, the Massachusetts framework includes an introduction to simple combinatorial knowledge.

Findings

The findings, with respect to the percentage data and probability, item type, computational difficulty, and cognitive complexity, are summarized in Exhibit 30.

Exhibit 30. Summary of the Data and Probability Strand

Dimension	Massachusetts	Hong Kong
Total number of items	6	2
Percentage of total items	17%	6%
Item type	Five items are multiple choice and one is short closed constructed response	Both items are short closed constructed-response
Computational difficulty	All six items have low computational difficulty	Both items have low computational difficulty
Cognitive complexity	Three items are Level 1 and three are Level 2	One item is Level 1 and one Level 2+

- *Proportion of measurement items.* Data items account for only 6% of Hong Kong items, but 17% of the Massachusetts assessment.
- *Item type.* The Hong Kong data items continue to emphasize constructed response, with all problems requiring students to construct an answer, compared with 1 out of 6, or 17%, of the Massachusetts items.
- *Computational difficulty.* Neither Hong Kong nor Massachusetts data items involve more than very simple arithmetic, usually numbers from 0 to 10.
- *Cognitive complexity.* The percentage of problems with middle or high cognitive demand was 50% on the Hong Kong assessment and 50% on the Massachusetts assessment.

Thus, Massachusetts stresses data more than Hong Kong, but Hong Kong's problems are as cognitively demanding.

Constructing graphs—Massachusetts and Hong Kong items both involve the construction of graphs from given data (Exhibit 31)

The two graph constructions in Exhibit 31 illustrate straightforward data tasks. The Massachusetts item presents the raw data in a table, and the Hong Kong item starts with a tally chart requiring students to construct the frequencies in numbers. However, the Massachusetts construction is somewhat more complicated in that students have to identify and aggregate cases with the same outcomes, whereas the Hong Kong construction requires simply adding the counted tallies in each category to a partially completed picture graph.

Exhibit 31. Data and Combinations—Constructing Graphs

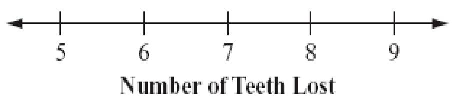
Massachusetts

Noah asked eight of his friends, “How many teeth have you lost?” The chart shows their answers.

Number of Teeth Lost

Name of Friend	Number of Teeth Lost
Alex	6
Jennifer	8
Kyle	7
Cody	5
Amanda	8
Sammy	7
Dan	9
Sarah	8

Noah started to make the line plot below to show the data from his chart. Put **X**s above the correct numbers to complete the line plot.



Hong Kong

Miss Wong did a survey on the participation of P.3B pupils in the activities for Games Day.

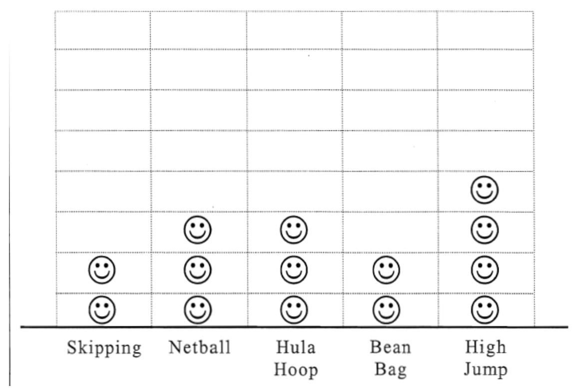
(a) According to the records, complete the table below.

Activities	Skipping	Netball	Hula Hoop	Bean Bag	High Jump
Records	卌 I	III	卌	IIII	卌 II
Number of pupils					

(b) According to the above data, complete the pictogram below.

The Participation of P.3B Pupils on Games Day

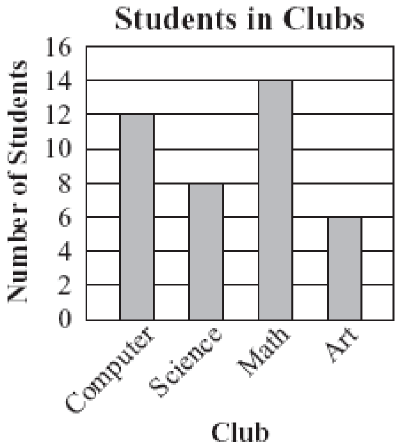
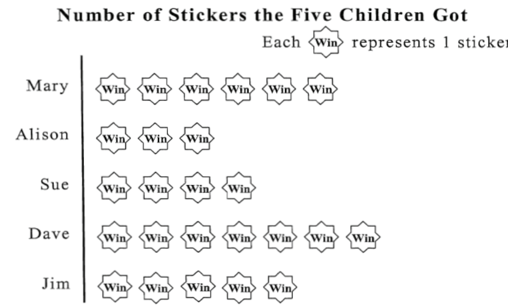
Each ☺ represents 1 pupil



Interpreting graphs—Massachusetts and Hong Kong items both involve multistep problems (Exhibit 32)

A Massachusetts item, shown in Exhibit 32, involves a “how many more than” question wherein students first read the data from a bar graph and then compute the difference between two of the categories. The comparison Hong Kong item displays a simpler pictograph that shows the frequencies for each child of winning a sticker. However, the Hong Kong item poses a series of three questions that require students to interpret the graph from different perspectives. The first two questions require calculating the maximum difference in stickers won and the number of children who won more than a given amount. The last question is a nonroutine problem that requires students to extend beyond the information directly presented in the chart to identify how many more children could potentially win six stickers if they played two more games.

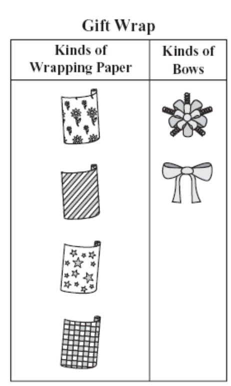
Exhibit 32. Data and Combinations—Interpreting Graphs

Massachusetts	Hong Kong
The bar graph below shows the number of students who belong to each club at Patterson School.  How many **more** students belong to the Math Club than to the Art Club? a. 4 b. 6 c. 8 d. 14	Five children went to a fun fair. A sticker was given to each winner of a game. The pictogram below shows the number of stickers they got.  (a) The child who won the greatest number of games got ________ more stickers than the child who won the least number of games. (b) The children who got 6 stickers or more were given a gift. According to the pictogram above, the child/children who got a gift was/were ______________________________ . (c) If each child could play two more games, the child/children who still had a chance of getting a gift was/were ______________________________ .

Combinations—Massachusetts items involve combinatoric understandings (Exhibit 33)

A Massachusetts item, shown in Exhibit 33, requires basic knowledge of constructing combinations given two different groups of items. The Hong Kong assessment includes no combinatoric items.

Exhibit 33. Data and Combinations—Combinations

Massachusetts	Hong Kong
Mandy is going to wrap a gift. The kinds of wrapping paper and bows she can choose are shown below.  How many different ways can Mandy choose 1 kind of wrapping paper and 1 kind of bow?	**The Hong Kong test includes no combinations items.**

Summary

Data and probability items are given greater emphasis on the Massachusetts assessment than on the Hong Kong assessments. However, the problems are of roughly equal cognitive challenge.

VI. ALGEBRA

Context

The core of elementary school algebra comprises using letters to represent unknown numbers, creating and solving number sentences, extending and generalizing patterns, and developing a sense of equality as equivalence and not as an operation. The Hong Kong BCA framework does not include a section on algebra at Grade 3, and algebra also is not included as a separate TIMSS-4 content area, as it is on TIMSS-8.

The Massachusetts Grade 3 framework differs from the Hong Kong framework and TIMSS in that it features items termed "algebra." Specifically, the Massachusetts Grade 3 framework includes a strand devoted to *Patterns, Relations, and Algebra* consisting of four elements:

- Create, describe, extend, and explain symbolic (geometric) patterns and addition and subtraction patterns, for example, 2, 6, 10, …; and 50, 45, 40 …
- Determine which symbol (<, >, or =) is appropriate for a given number sentence, e.g., 7×8 .?. $49 + 6$.
- Determine the value of a variable (through 10) in simple equations involving addition, subtraction, or multiplication, e.g., $2 + \_ = 9$; $5 \times \nabla = 35$.
- Write number sentences using +, –, ×, ÷, <, =, and/or > to represent mathematical relationships in everyday situations.

Altogether the Massachusetts framework identifies seven items under the algebra domain (Appendix 1); five are items involving number sentences that Massachusetts classifies as *Patterns, Relations, and Algebra*. However, three of these items involve the use of the appropriate equality or inequality sign and do not require an understanding of number sentences. For this study, we have included these three items within the number strand, along with Hong Kong assessment items involving similar types of operations.

Findings

The four remaining Massachusetts assessment items under algebra have the following characteristics:

- *Proportion of algebra items.* Algebra items represent one fifth of all Massachusetts Grade 3 test items if one uses the Massachusetts classification, but only one ninth of the total test items under our reclassification.
- *Item type.* Three of the four algebra items are multiple-choice items; however, the remaining pattern item is one of only two open constructed-response items on the Massachusetts test.
- *Computational difficulty.* All four algebra items require only a low level of computation.
- *Cognitive complexity.* Three of the four algebra items have low cognitive demand.

Patterns—Massachusetts items involve completing and extending patterns (Exhibit 34)

Two Massachusetts algebra items, shown in Exhibit 34, involve numeric patterns and were considered sufficiently unique to exclude them from the numbers category. The first item involves a pattern that follows a simple subtract 8 rule, with the subtraction rule stated in the item. The second item involves a repeating pattern of objects, in this case a banana followed by three oranges. The problem requires two steps: first to complete the pattern and then a simple computation (counting or multiplication) to compute the number of oranges used.

Exhibit 34. Patterns, Relations and Algebra—Patterns

<table>
<tr><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>Ms. Mackey wrote the number pattern below using the rule “subtract 8.”
187, 179, 171, __?__, 155, 147, 139
What is the missing number in Ms. Mackey’s pattern?
a. 163 b. 168 c. 170 d. 177</td><td rowspan="3">The Hong Kong test includes no algebra pattern items.</td></tr>
<tr><th>Massachusetts</th></tr>
<tr><td>Zoey is using bananas and oranges to make the pattern shown below. The rule for her pattern is ABBB.

Zoey will follow the rule for her pattern a total of 4 times.
How many oranges will Zoey use in all? Show or explain how you got your answer?</td></tr>
</table>

Number sentences—Massachusetts items involve an understanding of equivalent number sentences (Exhibit 35)

The two Massachusetts algebra items that involve number sentences that were not considered to be actually number strand items are shown in Exhibit 35 and require an understanding of equivalent number sentences. The first item implicitly assesses an understanding of the additive and multiplicative identities as students recognize that 5 + 0 is equivalent to 5 x 1. The second item involves an understanding of the inverse relationship between division and multiplication. Despite the algebraic concepts involved, neither item is computationally difficulty or cognitively complex given their computational nature.

Exhibit 35. Patterns, Relations and Algebra—Number Sentences

<table>
<tr><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>Which number sentence is true?

Ⓐ $5 + 0 = 5 \times 1$
Ⓑ $5 + 1 = 5 \times 1$
Ⓒ $5 + 0 = 5 \times 0$
Ⓓ $5 + 1 = 5 \times 0$</td><td rowspan="3">The Hong Kong test includes no algebra number sentence items.</td></tr>
<tr><th>Massachusetts</th></tr>
<tr><td>Candace wrote the number sentence below.

$15 \div 3 = \square$

Which of these is another way to write Candace’s number sentence?

Ⓐ $15 + \square = 3$
Ⓑ $15 \times \square = 3$
Ⓒ $3 + \square = 15$
Ⓓ $3 \times \square = 15$</td></tr>
</table>

Summary

A strand on patterns, relations, and algebra is included in the Massachusetts assessment, but not in the Hong Kong assessment or TIMSS. However, the dominant type of problem consists of using number sentences to determine equality or greater or less than, which overlaps with the numbers domain. The pattern problems are straightforward, and the number sentence problems focus on equivalent number sentences.

VII. Summary Comparison and Implications

This concluding section summarizes the differences in problem characteristics between the Massachusetts and Hong Kong Grade 3 assessment items by strand emphasis, item type, computational difficulty, and cognitive complexity. These differences suggest a number of explanations for Hong Kong’s world-class TIMSS-4 performance, which exceeds the performance of Massachusetts, the best in the United States.

1. *Hong Kong's Grade 3 assessment items emphasize the number and measurement strands (75% of the items), which form the core of a strong foundation in early mathematics understanding.* This focus reflects the need for a strong base of knowledge in number concepts and measurement, essential for doing more-advanced work in fractions and algebra. Further, a solid understanding of measurement topics, including length, volume, money, weight, and maps, is critical to handling real-world mathematics applications, including later work in geometry.

   By contrast, the Massachusetts assessment includes 11% of its items in the patterns, relations, and algebra domain, which Hong Kong omits, and 17% of its items in the data analysis, statistics, and probability domain, for which Hong Kong only presents two items (6%). The U.S. National Mathematics Advisory Panel report supports the Hong Kong assessment's downplaying of patterns, which the panel believes is overemphasized in the early U.S. mathematics curriculum. However, the relatively limited Hong Kong focus on data is consistent with Hong Kong's lower relative performance in this content area.

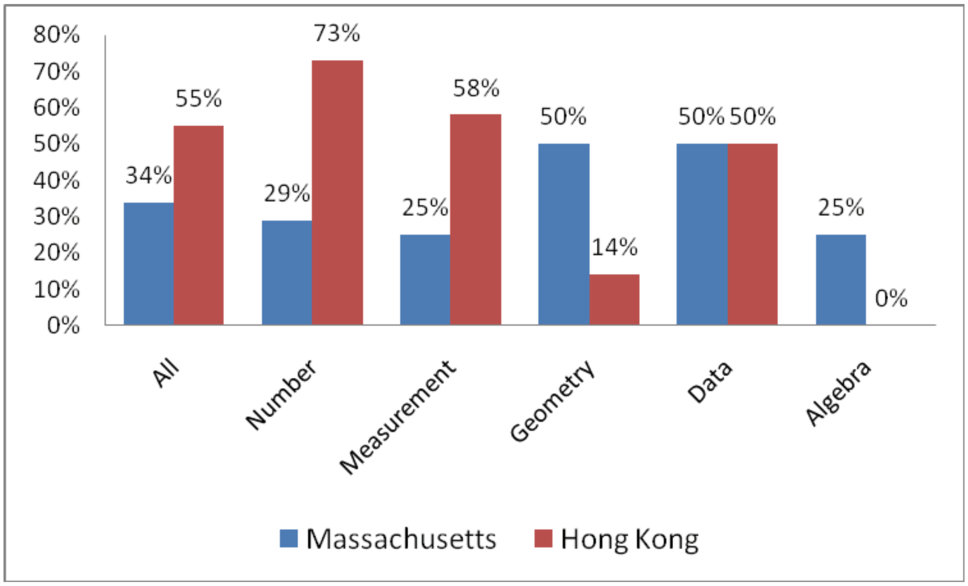
2. *Hong Kong items are more likely to require students to construct a response (86%) than are Massachusetts items (29%).* Constructed-response items are more demanding in that students must generate the correct answer working by completely through the problem, without the advantage of being able to choose a correct answer from a list.

3. *The Hong Kong assessment contains a somewhat higher percentage (71%) of items with graphical content than the Massachusetts assessment, at 57%.* Graphics in items may present an additional representation or nonverbal setting that may provide better access to the item or may support real-world situations that require students to interpret or identify key problem features or that may require students to translate concrete representations into mathematical structures.

4. *The Hong Kong assessment has 37% of its items requiring more than simple computational difficulty, compared with only 3% for the Massachusetts assessment.* In the numbers domain, where computation is an integral component of the solution, 13 out of 15 (87%) of Hong Kong items were of higher computational difficulty, whereas only 1 out of 17 (6%) of the Massachusetts items in numbers assessed more than simple computational skills. A study of high-performing countries on TIMSS and PISA found that countries with high performance on items that were cognitively demanding were also high performers on procedural computational problems (Ginsburg, Leinwand, Cooke, Noell, & Pollock, 2005).

5. *The percentage of Hong Kong items with middle or high cognitive complexity was 55%, or two thirds more than the 34% of middle or high cognitive demand items on the Massachusetts assessment (Exhibit 36).* Focusing only on the numbers and measurement domains shows that 18 out of 27 items (68%) on the Hong Kong assessment were of middle or high cognitive complexity, compared with only 6 out of 21 (29%) for the Massachusetts assessment.

Exhibit 36. Percentage of Items Requiring More Than Simple Recall of Concepts, by Mathematical Strand



Features That Make Items Challenging

The higher cognitive difficulty level of the items on the Hong Kong assessment, especially in the core areas of numbers and measurement, is a distinguishing feature of the comparison with the Massachusetts assessment. It is important that assessment problems establish expectations for students' deep conceptual understanding and students' ability to develop strategies for applying conceptual learning even in the very early years of mathematics learning.

An examination of the items rated as more cognitively demanding, most from the Hong Kong assessment, shows that the following features increase mathematical rigor:

- *Developing multistep solutions that require students to carry out a series of mathematical procedures rather than a solution with single calculation or one-step mathematical analysis.*
  Example: A Hong Kong money problem involves translating a word price into numerals, reading a bill showing the amount paid, and then making change by identifying the needed combination of Hong Kong currency (Exhibit 23).

- *Solving problems in nonroutine situations that require students to adapt what they have learned from familiar situations.*
  Example: A Hong Kong division problem presents students with a worked-out long division problem that has missing numbers in the dividend, intermediate calculations, and the answer, thereby requiring them to fill in the blanks (Exhibit 11).

- *Satisfying multiple problem conditions simultaneously.*
  Example: A Hong Kong problem of ordering digits to find the smallest number adds a twist that the number must also be odd (Exhibit 7).

- *Correctly differentiating among multiple representations of the same concept.*
  Example: A Hong Kong fraction problem requires students to identify the relationship between pairs of proper fractions with equal numerators, equal denominators, equal numbers in numerator and denominator, and whole numbers (Exhibit 10).

- *Translating different representations to a common representation before completing solution.*
  Example: A Hong Kong ordering of numbers problem requires ordering numbers expressed three different ways: symbols on a place-value chart, words, and numerals (Exhibit 8).

- *Finding the most efficient solution strategy among alternative solution strategies.*
  Example: A Hong Kong number subtraction problem is a complicated negative number computation if operations are carried out sequentially but can be simplified if the addition is done out of order prior to the subtraction (Exhibit 9).

- *Having to manipulate problem elements to obtain the solution.*
  Example: A Massachusetts geometry problem requires students to select appropriate pieces and rotate them so that, when combined, they make up a composite figure (Exhibit 26).

- *Selecting the appropriate information from a set that includes extraneous information.*
  Example: A Hong Kong geometry problem requires the matching of three-dimensional geometric figures to their names and provides some figures for which no match is included (Exhibit 27).

These features of more challenging problems, drawn primarily but not entirely from the Hong Kong assessment, demonstrate that even at Grade 3, the teaching of mathematics must be far more than the mechanical application of definitions and formulas. In top-performing Hong Kong, the assessment items often require students to demonstrate deep conceptual understanding and the capacity to apply foundational mathematical concepts in multistep, real-world situations.

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APPENDIX 1: MASSACHUSETTS MATHEMATICS FRAMEWORK

Numbers: ***Students engage in problem solving, communicating, reasoning, connecting, and representing as they:***

3.N.1 Exhibit an understanding of the values of the digits in the base ten number system by reading, modeling, writing, comparing, and ordering whole numbers through 9,999.

3.N.2 Represent, order, and compare numbers through 9,999. Represent numbers using expanded notation (e.g., 853 = 8 x 100 + 5 x 10 + 3), and written out in words (e.g., eight hundred fifty-three).

3.N.3 Identify and represent fractions (between 0 and 1 with denominators through 10) as parts of unit wholes and parts of groups. Model and represent a mixed number (with denominator 2, 3, or 4) as a whole number and a fraction, e.g., $1\frac{2}{3}$, $3\frac{1}{2}$.

3.N.4 Locate on the number line and compare fractions (between 0 and 1 with denominators 2, 3, or 4, e.g., $\frac{2}{3}$).

3.N.5 Recognize classes to which a number may belong (odd numbers, even numbers, and multiples of numbers through 10). Identify the numbers in those classes, e.g., the class of multiples of 7 between 1 and 29 consists of 7, 14, 21, 28.

3.N.6 Select, use, and explain various meanings and models of multiplication (through 10 □10). Relate multiplication problems to corresponding division problems, e.g., draw a model to represent 5 x 6 and 30 ÷ 6.

3.N.7 Use the commutative (order) and identity properties of addition and multiplication on whole numbers in computations and problem situations, e.g., 3 + 4 + 7 = 3 + 7 + 4 = 10 + 4.

3.N.8 Select and use appropriate operations (addition, subtraction, multiplication, and division) to solve problems, including those involving money. This standard is intentionally the same as standard 4.N.10.

3.N.9 Know multiplication facts through 10 x 10 and related division facts, e.g., 9 x 8 = 72 and 72 ÷ 9 = 8. Use these facts to solve related problems, e.g., 3 x 5 is related to 3 x 50.

3.N.10 Add and subtract (up to four-digit numbers) and multiply (up to two-digit numbers by a one-digit number) accurately and efficiently.

3.N.11 Round whole numbers through 1,000 to the nearest 10, 100, and 1,000.

3.N.12 Understand and use the strategies of rounding and regrouping to estimate quantities, measures, and the results of whole-number computations (addition, subtraction, and multiplication) up to two-digit whole numbers and amounts of money to $100, and to judge the reasonableness of the answer.

3.N.13 Use concrete objects and visual models to add and subtract (only when the answer is greater than or equal to zero) common fractions (halves, thirds, fourths, sixths, and eighths) with like denominators.

Measurement: ***Students engage in problem solving, communicating, reasoning, connecting, and representing as they:***

3.M.1 Demonstrate an understanding of the attributes length, area, and weight, and select the appropriate type of unit for measuring each attribute using both the U.S. Customary (English) and metric systems.

3.M.2 Carry out simple unit conversions within a system of measurement, e.g., hours to minutes, cents to dollars, yards to feet or inches, etc.

3.M.3 Identify time to the minute on analog and digital clocks using a.m. and p.m. Compute elapsed time, using a clock for times less than one hour (i.e., minutes since), and using a calendar (e.g., days since).

3.M.4 Estimate and find area and perimeter of a rectangle, using diagrams and grids, or by measuring.

3.M.5 Identify and use appropriate metric and U.S. Customary (English) units and tools (e.g., ruler, scale, thermometer, clock) to estimate, measure, and solve problems involving length, area, weight, temperature, and time.

Geometry: ***Students engage in problem solving, communicating, reasoning, connecting, and representing as they:***

3.G.1 Compare and analyze attributes and other features (e.g., number of sides, corners, diagonals, and lines of symmetry) of two-dimensional geometric shapes.

3.G.2 Describe, model, draw, compare, and classify two-dimensional shapes, e.g., circles, triangles, and quadrilaterals. Identify and describe simple three-dimensional shapes, e.g., cubes, spheres, and pyramids.

3.G.3 Identify angles as right angles, less than a right angle, and greater than a right angle.

3.G.4 Identify and draw parallel lines, perpendicular lines, and other intersecting lines.

3.G.5 Using ordered pairs of whole numbers and/or letters, locate and identify points on a grid.

3.G.6 Identify and draw lines of symmetry in two-dimensional shapes.

3.G.7 Predict and explain the results of taking apart and combining two-dimensional shapes.

Data Analysis, Statistics and Probability: ***Students engage in problem solving, communicating, reasoning, connecting, and representing as they:***

3.D.1 Collect and organize data using observations, measurements, surveys, or experiments, and identify appropriate ways to display the data.

3.D.2 Match representations of a data set in the forms of tables, line plots, pictographs, tallies, or bar graphs with the actual data set.

3.D.3 Construct and draw conclusions from representations of data sets in the forms of tables, line plots, pictographs, tallies, and bar graphs.

3.D.4 List and count the number of possible combinations of objects from two sets, e.g., how many different outfits can one make from a set of two sweaters and a set of three skirts?

Patterns, Relations and Algebra: ***Students engage in problem solving, communicating, reasoning, connecting, and representing as they:***

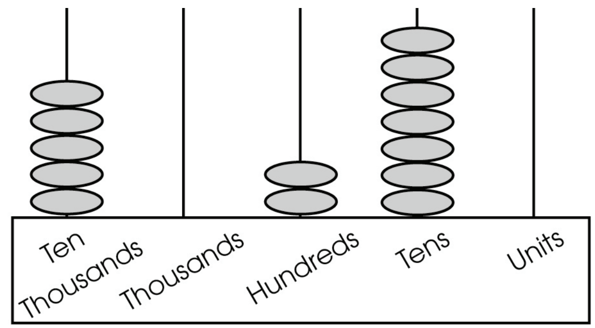
3.P.1 Create, describe, extend, and explain symbolic (geometric) patterns and addition and subtraction patterns, e.g., 2, 6, 10, …; and 50, 45, 40.…

3.P.2 Determine which symbol (<, >, or =) is appropriate for a given number sentence, e.g., 7×8 .?. $49 + 6$.

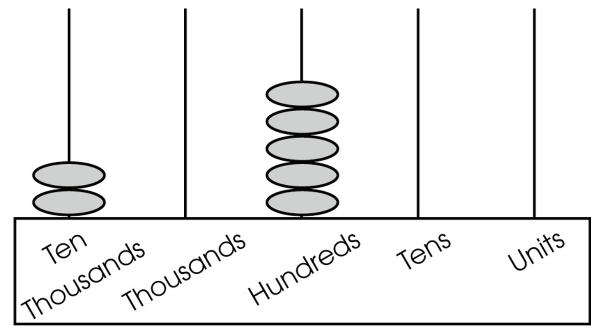
3.P.3 Determine the value of a variable (through 10) in simple equations involving addition, subtraction, or multiplication, e.g., $2 + \_ = 9$; $5 \times \nabla = 35$.

3.P.4 Write number sentences using +, –, ×, ÷, <, =, and/or > to represent mathematical relationships in everyday situation

APPENDIX 2: ITEM-BY-ITEM COMPARISON OF MASSACHUSETTS GRADE 3 VS. HONG KONG GRADE 3

Topic: Number—Ordering Whole Numbers		
	Massachusetts	**Hong Kong**
Item	The table below shows how many books three classes read. **Books Read** **Class** — **Number of Books** **Ms. Potter** — **1023** **Ms. Hogan** — **?** **Mr. Garcia** — **1067** Ms. Hogan's class read more books than Ms. Potter's class and fewer books than Mr. Garcia's class. Which of these could be the number of books Ms. Hogan's class read? a. 1074 b. 1166 c. 1005 d. 1062	Arrange the following numbers from the largest to the smallest (give the answers **in numerals.)** a.  b. Fifty-two thousand and seven c. 50720 ______________,. ___________, ____________ (Largest) (Smallest)
Item Format	Multiple choice	Short closed constructed-response
Computational Difficulty	Low (4-digit numbers with ordering in tens and ones)	Medium (5-digit numbers with ordering in thousands and hundreds)
Cognitive Complexity	Level 1 (one-step place value understanding to identify the number between two given numbers)	Level 2 (multistep place value understanding involving converting and ordering three different representations of numbers in an open-ended situation)
Comments	Context adds to reading load, but table presents data accessibly.	No context, but numbers are presented in three different representations (picture, words, symbols) that must all be converted to numerals. Picture reinforces place value understanding.

Topic: Number—Ordering Whole Numbers Using Number Sentences and Symbols			
	Massachusetts		Hong Kong
Item	Which symbol belongs in the circle below to make a true number sentence? $7 \times 7 \bigcirc 34 + 13$ a. > b. – c. < d. =	The Hamilton family drove 138 miles. The Jefferson family drove 206 miles. Which of these correctly compares the number of miles each family drove? a. 138 < 206 b. 138 + 206 c. 138 = 206 d. 138 > 206	
Item Format	Multiple choice	Multiple choice	
Computational Difficulty	Low (two basic one-step calculations)	Low (only requires identification of correct sign for order of 2 three-digit numbers)	
Cognitive Complexity	Level 1 (requires two calculations and identification of correct order sign)	Level 1 (one-step comparison and identification of correct order sign)	
Comments	Combines number and algebra strand with use of number sentence. Massachusetts classifies this as Patterns, Relations and Algebra. We have classified it as Number.	Context of two distances; students must identify correct order sign. Combines number and algebra strands.	

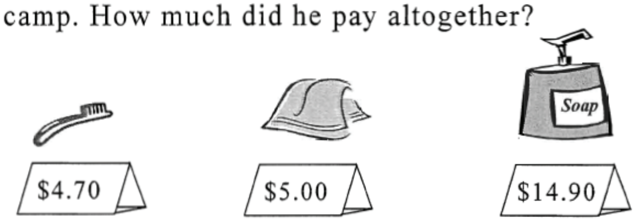
Topic: Number—Rounding Whole Numbers		
	Massachusetts	**Hong Kong**
Item	What is 972 rounded to the nearest ten? a. 900 b. 970 c. 980 d. 1000	 Which of the following numbers is nearest to the numbers shown on the abacus? a. 19000 b. 19500 c. 21000 d. 22500
Item Format	Multiple choice	Multiple choice
Computational Difficulty	Low (three-digit number rounded to nearest 10)	Medium (identification of five-digit number closest to 20,500)
Cognitive Complexity	Level 1 (procedural, one-step problem requiring rounding down to the nearest 10)	Level 2 (requires using an “abacus” representation to identify 20,500 and using place value or rounding understandings to determine which of four numbers is closest)
Comments	Standard, context-free rounding problem	More conceptual “which number is closest” situation with the need to translate the “abacus” representation

Topic: Number—Place Value		
	Massachusetts	**Hong Kong**
Item	*Use the number tiles from your tool kit to answer the following question.* Alan has the number tiles shown below. **4 | 7 | 8 | 1** a. Use all of Alan's number tiles to make the four-digit number with the **smallest** value. Use each number tile only one time. Write the number in the boxes below. ☐☐☐☐ b. What is the value of the digit 7 in the number you made? Explain your answer.	**3 | 0 | 8 | 2 | 5** Arrange the 5 number cards above to form the smallest 5-digit **odd** number. Answer: ☐☐☐☐☐ **NOTE: This item is from an alternate form of the test and is not included in Exhibits 3 or 6.**
Item Format	Short closed and open constructed-response	Short closed constructed-response
Computational Difficulty	Low (creating the smallest number given four digits and identifying the value of the digit in the tens place)	Low (creating the smallest number given five digits with the constraint of it being "odd")
Cognitive Complexity	Level 2 (multistep open-ended problem requiring construction of the smallest four digit number)	Level 2+ (multistep open-ended problem requiring construction of the smallest five-digit number; "odd" adds nuance)
Comments	A digit arrangement task requiring students to create the smallest four-digit number given four different digits and then to identify the value of the 7 (70 in 1478). Actual number tiles are available as concrete materials and **smallest** is bold. The correct answer to part b. is dependent upon at least a partially correct answer to part a.	A digit arrangement task requiring students to create the smallest odd five-digit number given five different digits. "Odd" adds interesting reasoning twist (must end in 5, the largest odd digit, so 02385). Actual number cards are **not** available and **odd** is bold and underlined.

Topic: Number—Addition and Subtraction With Number Sentences			
	Massachusetts	**Hong Kong**	
Item		a. 576 + 282 = ___________ b. 340 – 500 + 460 = ___________	
Item Format		Item a. Short closed constructed-response	Item b. Short closed constructed-response
Computational Difficulty		Medium (three-digit numbers with one regrouping)	Low (becomes 800 – 500)
Cognitive Complexity		Level 1 (three-step procedure for adding)	Level 2+ (multistep; students encounter a negative situation if they compute left to right)
Comments		Requires finding a sum with one regrouping.	Requires finding a difference and a sum by adding first and then subtracting with no regrouping.

Topic: Number—Subtraction		
	Massachusetts	**Hong Kong**
Item	Neva has 16 pencils in her desk. Tracy has 8 pencils in her desk. Which number sentence can be used to find how many more pencils Neva has than Tracy? a. 8 – 16 = _____ b. 8 + 16 = _____ c. 16 + 8 = _____ d. 16 – 8 = _____	850 – 133 – 409 = ○ A. 717 ○ B. 1 126 ○ C. 318 ○ D. 308
Item Format	Multiple choice	Multiple choice
Computational Difficulty	Low (no computation, just understanding of subtraction and correct order of the numbers)	Medium (three-digit subtraction with regrouping)
Cognitive Complexity	Level 1 (basic translation of situation into one-step number sentence.	Level 2 (multistep subtraction involving regrouping)
Comments	Uses a context to assess the meaning of subtraction and identification of the correct number sentence.	Requires two subtractions, both with regrouping, or an addition and a subtraction with regrouping.

<table>
<tr><th colspan="3">Topic: Number—Subtraction Word Problems</th></tr>
<tr><th></th><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>Item</td><td>Mr. Wilson’s class made the chart below to show the number of birds that ate at a bird feeder on five days.
Birds That Ate at the Bird Feeder<table><tr><th>Day</th><th>Number of Birds</th></tr><tr><td>Monday</td><td>8</td></tr><tr><td>Tuesday</td><td>18</td></tr><tr><td>Wednesday</td><td>30</td></tr><tr><td>Thursday</td><td>12</td></tr><tr><td>Friday</td><td>20</td></tr></table>How many more birds ate at the bird feeder on Wednesday than on Monday?
Write your answer in the Answer Box below.</td><td>Ben has 307 stamps and Sally has 126 stamps. Ben has ___________ more stamps than Sally has.</td></tr>
<tr><td>Item Format</td><td>Short closed constructed-response</td><td>Short closed constructed-response</td></tr>
<tr><td>Computational Difficulty</td><td>Low (subtraction of a one-digit from a two-digit number, but simple enough to be done mentally)</td><td>Medium (subtraction of 2 three-digit numbers with regrouping and a medial zero)</td></tr>
<tr><td>Cognitive Complexity</td><td>Level 2 (retrieve data from data, identify two correct numbers and subtract within a problem context)</td><td>Level 1 (use a procedure to find a difference)</td></tr>
<tr><td>Comments</td><td>Requires the reading of data from a table. Context adds reading load, but table presents data accessibly. Requires students to understand that “how many more” implies subtraction. More is bold for emphasis. (Coded as data, statistics and probability by Massachusetts)</td><td>Requires students to understand that “how many more” implies subtraction. More is not bolded.</td></tr>
</table>

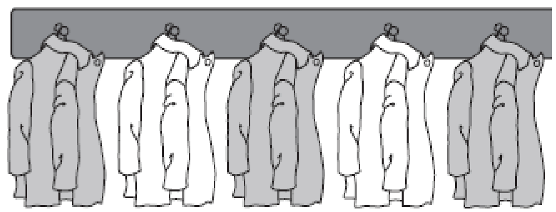
Topic: Number—Addition/Subtraction Word Problems			
	Massachusetts	**Hong Kong**	
Item	**OTHER THAN THE "BIRDS THAT ATE AT THE BIRD FEEDER" SUBTRACTION PROBLEM ON THE PREVIOUS PAGE, MASSACHUSETTS DOES NOT INCLUDE ANY ONE- OR TWO-STEP ADDITION OR SUBTRACTION STORY PROBLEMS.**	a. There were 870 boxes of fruit juice in the warehouse. The works move 351 boxes away **yesterday** and they moved the same number of boxes away **today**. How many boxes of fruit juice are left in the warehouse? (Show your work) b. My brother joined a summer camp. He bought a toothbrush, a towel and a bottle of soap before going to the camp. How much did he pay altogether?  Answer: He paid ________ dollars and ________ cents altogether.	
Item Format		Item a. Open constructed-response	Item b. Short closed constructed-response
Computational Difficulty		Medium (three-digit subtraction with regrouping)	Medium (three- and four-digit money amounts with regrouping)
Cognitive Complexity		Level 2 (multistep subtraction involving regrouping within a problem situation)	Level 2 (use a procedure to find the sum of three money amounts within a problem situation)
Comments		Requires students to understand that the context calls for subtraction. Requires finding two successive differences (unless students double the 351 first) and includes regrouping.	Requires students to understand that the context calls for addition and requires finding the sum of three money amounts. The graphic for the addition problem reduces the reading load.

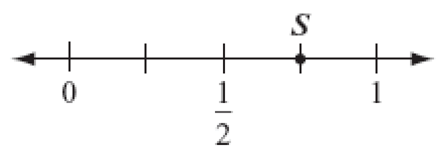
Topic: Number—Multiplication		
	Massachusetts	**Hong Kong**
Item	Compute: 83 × 4 Write your answer in the Answer Box below.	**209 × 3 = __________**
Item Format	Short closed constructed-response	Short closed constructed-response
Computational Difficulty	Low (two-digit by one-digit multiplication with one regrouping)	Medium (three-digit by one-digit multiplication with one regrouping and a medial zero)
Cognitive Complexity	Level 1 (use a procedure to find a product)	Level 1+ (use a procedure to find a product, but medial zero in third grade adds nuance)
Comments		

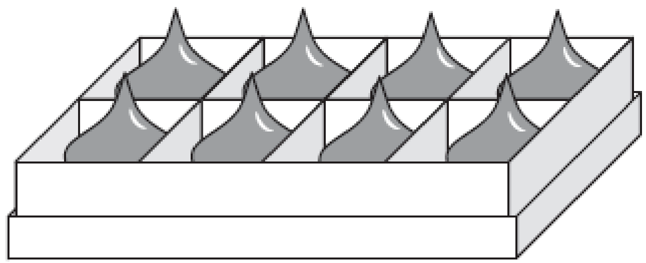
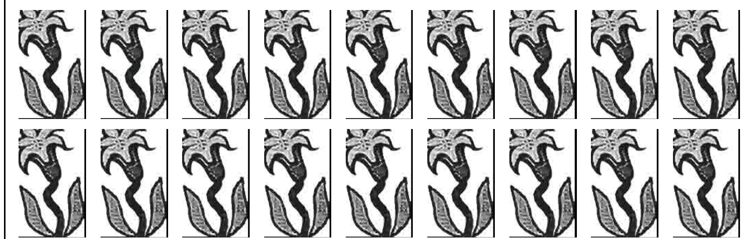
Topic: Number—Multiplication		
	Massachusetts	**Hong Kong**
Item	Cindy wrote the number sentence below. $\boxed{?} \times 3 = 24$ In the Answer Box below, write the missing number that makes Cindy's number sentence true. $\boxed{}$	$188 + 78 \times 4 =$ __________
Item Format	Short closed constructed-response	Short closed constructed-response
Computational Difficulty	Low (requires knowledge of division fact 24 ÷ 3, or what times 3 is 24)	Medium (multiplying with regrouping and then adding 2 three-digit numbers with two regroupings)
Cognitive Complexity	Level 1 (one-step finding an unknown)	Level 2+ (two-step, two difference operation that requires an understanding of order of operations)
Comments	Combines number and algebra strand with use of number sentence. Massachusetts classifies this as Patterns, Relations and Algebra. We have classified it as Number.	Students must know to multiply first and then add.

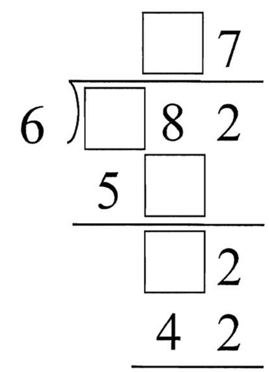
Topic: Number—Multiplication			
	Massachusetts		**Hong Kong**
Item	a. Jenny collected 10 seashells. She collected 2 times as many seashells as Beth collected. How many seashells did Beth collect? a. 5 b. 8 c. 12 d. 20 b. Missy wants to put 12 stickers on her paper. What is one way that she can put 12 stickers on her paper? a. 3 rows of 3 stickers b. 3 rows of 6 stickers c. 4 rows of 2 stickers d. 4 rows of 3 stickers		Dad bought a refrigerator and paid for it in 8 monthly installments of $496 each month. Altogether he paid $ _______ for the refrigerator.
Item Format	Item a. Multiple choice	Item b. Multiple choice	Short closed constructed-response
Computational Difficulty	Low (simple number fact 10 × 2)	Low (identify 4 × 3 as only context that results in 12)	Medium (three-digit by one-digit multiplication with three regroupings)
Cognitive Complexity	Level 1 (one-step recall with context that requires understanding that multiplication or division is the appropriate operation)	Level 1 (one-step identification of two factors of 12)	Level 2 (use a procedure to find a product within a problem situation)
Comments	Uses word “times,” which makes it even easier.	Rows and stickers provide helpful context.	Problem requires students to identify multiplication as the appropriate operation.

Topic: Number—Division			
	Massachusetts	Hong Kong	
Item	Which symbol belongs in the circle below to make the number sentence true? 45 ÷ 9 O 35 ÷ 7 a. < b. = c. > d. +	a. Use short division to calculate 828 ÷ 4. $4\overline{)828}$ b. My brother saved $624 in the past 3 years. On average how much did he save each year? (show you work)	
Item Format	Multiple choice	Item a. Short closed constructed-response	Item b. Open constructed-response
Computational Difficulty	Low (two basic one-step calculations and a comparison)	Medium (three-digit by one-digit division with a medial zero in the quotient)	Medium (three-digit by one-digit division with a medial zero in the quotient)
Cognitive Complexity	Level 1 (requires two calculations and identification of correct order sign)	Level 2 (short division abbreviates the standard procedure)	Level 2 (students use a procedure to find a quotient, but must identify division as the appropriate operation)
Comments	Combines number and algebra strand with use of number sentence. Massachusetts classifies this as Patterns, Relations and Algebra. We have classified it as Number.		Requires an understanding that division is the appropriate operation.

Topic: Number—Fraction Concepts		
	Massachusetts	**Hong Kong**
Item	The coats shown below are hanging on coat hooks.  What fraction of the coats are white? Write your answer in the Answer Box below. ☐	Fill in the boxes with “$>$”, “$<$” or “$=$”. (a) $1 \;\square\; \frac{10}{10}$ (b) $\frac{1}{5} \;\square\; \frac{1}{8}$ (c) $\frac{3}{11} \;\square\; \frac{9}{11}$ (d) $\frac{2}{2} \;\square\; 2$
Item Format	Short closed constructed-response	Short closed constructed-response
Computational Difficulty	Low (correctly write fraction)	Low (insert correct order sign based on conceptual understanding of fractions)
Cognitive Complexity	Level 1 (identify fractional part of whole as 2 out of 5, or 2/5)	Level 2 (four difference items requiring students to identify two fractions (or whole numbers) as less than, greater than, or equal to each other)
Comments	Item uses appropriate representation to support basic concept of fraction.	Tasks assess higher-order fraction understandings, including 10/10 = 1, 1/a > 1/b if a < b, and a/11 < b/11 if a < b.

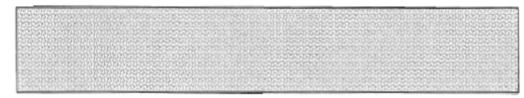
Topic: Number—Fraction Concepts		
	Massachusetts	**Hong Kong**
Item	Point S is shown on the number line below.  Which fraction best names point *S* on the number line? a. 1/4 b. 2/3 c. 3/4 d. 3/2	
Item Format	Multiple choice	
Computational Difficulty	Low (basic fraction concept without computation)	
Cognitive Complexity	Level 1 (identification of ¾ as fraction half-way between ½ and 1)	
Comments	Number line representation provides helpful support.	

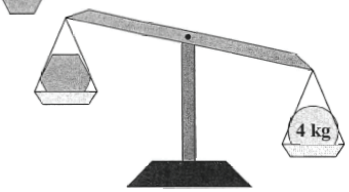
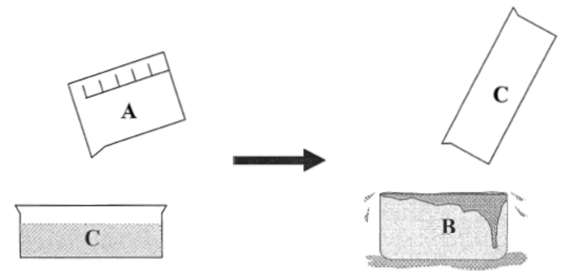
Topic: Number—Fraction Computation		
	Massachusetts	**Hong Kong**
Item	Marta and Nate had the box of 8 candies shown below.  Marta ate $\frac{5}{8}$ of the candies. Nate ate $\frac{2}{8}$ of the candies. Altogether, what fraction of the box of candies did they eat? a. 3/8 b. 7/8 c. 3/16 d. 7/16	Mum bought 18 flowers and gave 1/6 of them to a friend. She gave ______ flowers to her friend. 
Item Format	Multiple-choice	Short closed constructed-response
Computational Difficulty	Medium (procedural understanding that 5/8 + 2/8 =7/8)	Medium (procedural understanding that 1/6 of 18 is 3)
Cognitive Complexity	Level 2 (a two-step problem requiring students to add two fractions with like denominators within a problem situation)	Level 1 (a one-step problem requiring students to multiply a fraction by a whole number or to recognize that multiplying by 1/6 is the same as dividing by 6)
Comments	A graphic that supports the item and allows students to see and think 5 out 8 plus 2 out of 8 is 7 out of 8.	A graphic that supports the item and allows students to see that 1 out of 6 is three items or to create 6 groups of three flowers.

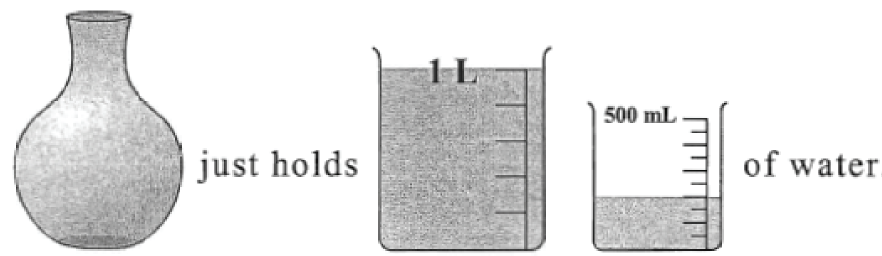
Topic: Number—Numerical Reasoning		
	Massachusetts	**Hong Kong**
Item	Maria is thinking of a number. The clues for her number are shown below: • It is a multiple of 5. • It is an even number • It is less than 18. Which of these could be Maria's number? a. 5 b. 20 c. 8 d. 10	Fill in the boxes with the correct numbers.  **NOTE: This item is from an alternate form of the test and is not included in Exhibits 3 or 6.**
Item Format	Multiple choice	Short closed constructed-response
Computational Difficulty	Low (two-digit numbers)	High (three-digit by one-digit long division)
Cognitive Complexity	Level 2 (requires identifying the number that satisfies three conditions and an understanding of "multiple," "even," and "less than")	Level 3 (requires an in-depth understanding of the division algorithm and significant reasoning skills (6 × _a_ 7 = _b_ 8 2 and _a_ × 6 = 5 __).
Comments		**Solution:** 17 × 6 = 102, 27 × 6 = 162, 37 × 6 = 222, **4**7 × 6 = 2**82**, but 4 × 6 cannot be 5 __. However, **9**7 × 6 is **5**82 and **9** × 6 = 5**4**. Alternatively, students might start with the lower box having to be a 4 since there is no remainder and then work backward to fill in the numbers

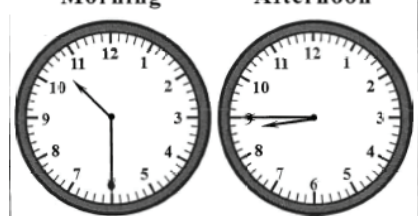
Topic: Number—Estimation		
	Massachusetts	**Hong Kong**
Item	Brianna bought 4 shirts. Each shirt cost \$8.95. Which estimate is closest to the total cost of the shirts that Brianna bought? a. \$32 b. \$36 c. \$38 d. \$40	**THERE ARE NO ESTIMATION ITEMS ON THE HONG KONG TEST.**
Item Format	Multiple choice	
Computational Difficulty	Low (round and multiply 4×9)	
Cognitive Complexity	Level 2 (two-step procedural application)	
Comments	Students must know that \$8.95 is almost \$9 and then correctly multiply 4×9.	

Topic: Measurement—Measurement Units		
	Massachusetts	**Hong Kong**
Item	Which unit can Sara use to measure the height of the snow in her backyard? a. gallon b. pound c. inch d. ounce	Fill in each of the following blanks with a suitable unit of weight. (a) A television set weighs about 20 ___________ . (b) A piece of cake weighs about 150 ___________ . (c) A bag of barbecue charcoal weighs about 5 ___________ .
Item Format	Multiple choice	Short closed constructed-response
Computational Difficulty	Low (no arithmetic)	Low (no arithmetic)
Cognitive Complexity	Level 1 (identify “inch” as unit of length)	Level 2 (identify unit that matches a given number of units for real objects)
Comments		Constructed-response format makes this much more difficult than an equivalent multiple-choice item.

Topic: Measurement—Length			
	Massachusetts	**Hong Kong**	
Item	**THERE ARE NO MEASURING LENGTH ITEMS ON THE MASSACHUSETTS TEST**	Mum and Dad want to buy a wardrobe. Which of the following measuring instruments is most suitable for measuring the height of the wardrobe? ○A.  Trundle Wheel ○B.  Ruler ○C.  Measuring Tape ○D.  String	Use a ruler to measure the length and the width of the ribbon below.  (a) The length of the ribbon is _______ cm. (b) The width of the ribbon is _______ mm.
Item Format		Multiple choice	Short closed constructed-response
Computational Difficulty		Low (no arithmetic required)	Low (no arithmetic required)
Cognitive Complexity		Level 1+ (identify most appropriate tool, but all four tools can measure length)	Level 1+ (use a ruler to measure length in cm and mm)
Comments			

Topic: Measurement—Measurement Concepts			
	Massachusetts	**Hong Kong**	
Item	**THERE ARE NO MEASUREMENT CONCEPTS ITEMS ON THE MASSACHUSETTS TEST**	The possible weight of  is  ◯ A. 3 kg ◯ B. 4 kg ◯ C. 5 kg ◯ D. 6 kg	 Container A is filled completely with water and all the water is poured into Container C. Then all the water in Container C is poured into Container B (the results are shown in the diagram above). Arrange Containers A, B and C in order, from the largest capacity to the smallest. Write the letters for the answer. Answer: ________ (Largest), ________, ________ (Smallest)
Item Format		Multiple choice	Short closed constructed-response
Computationa l Difficulty		Low (no arithmetic)	Low (no arithmetic)
Cognitive Complexity		Level 1 (identify weight less than 4 kg)	Level 3 (requires reasoning that A doesn't fit in C, so C > A, but the amount of water in A (now residing in C) will overflow B so that A > B.
Comments		Requires experience with a balance.	

Topic: Measurement—Conversions		
	Massachusetts	**Hong Kong**
Item	**THERE ARE NO MEASUREMENT CONVERSION ITEMS ON THE MASSACHUSETTS TEST**	 The capacity of the vase is ________ mL.
Item Format		Short closed constructed-response
Computational Difficulty		Low (adding 1000 + 200)
Cognitive Complexity		Level 2 (requires converting 1 L to 1000 mL and reading scale to get 200 mL, thereby applying understandings)
Comments		

Topic: Measurement—Time			
	Massachusetts	**Hong Kong**	
Item	The clock below shows the time that Mr. Stone put a cake in the oven.  a. The cake needs to bake for 30 minutes. At what time will the cake be done? b. Mr. Stone also wants to bake rolls. The rolls only need to bake for 10 minutes. At what time should Mr. Stone put the rolls in the oven so that they will be done at exactly the same time as the cake?	The clock stopped working because the battery in it ran out. The time when the clock stopped working was at ________ minutes to ________ . 	The clocks below show the time when Mr. Ho started and finished work on a certain day. Fill in the Working Hours Record Sheet for Mr. Ho using the '24-hour time'.  **Success Company** **Working Hours Record Sheet** Staff: Ho Ka Wa Start: ___ : ___ Finish: ___ : ___
Item Format	Short closed constructed-response	Short closed constructed-response	Short closed constructed-response
Computational Difficulty	Low (adding 30 minutes on a clock)	Low (no arithmetic)	Low (need to add 12 to get 24-hour time)
Cognitive Complexity	Level 2 (identifying a time 30 minutes later and 10 minutes earlier in problem situation)	Level 1+ (reading and writing time from a clock)	Level 2 (identifying time from a clock and converting it to 24-hour time)
Comments	The answer to part b. is not independent of the answer to part a.		8:45 being "afternoon" in a 10+ hour workday sends an interesting message.

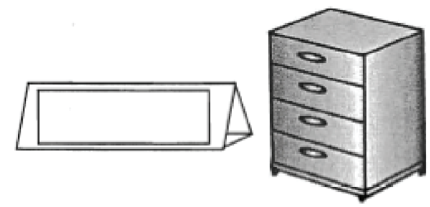
<table>
<tr><th colspan="3">Topic: Measurement—Calendar Time</th></tr>
<tr><th></th><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>Item</td><td>THERE ARE NO CALENDAR ITEMS ON THE MASSACHUSETTS TEST</td><td>
<table>
<tr><th colspan="7">December 2006</th></tr>
<tr><th>Sun</th><th>Mon</th><th>Tue</th><th>Wed</th><th>Thu</th><th>Fri</th><th>Sat</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>1</td><td>2</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td></tr>
<tr><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td></tr>
<tr><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr>
<tr><td>31</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
(a) Mum bought a washing machine on the 14th of December and arranged to have it delivered on the next Wednesday, which was the
_______ of ____________ .
(month)

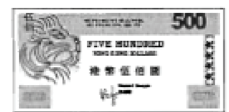
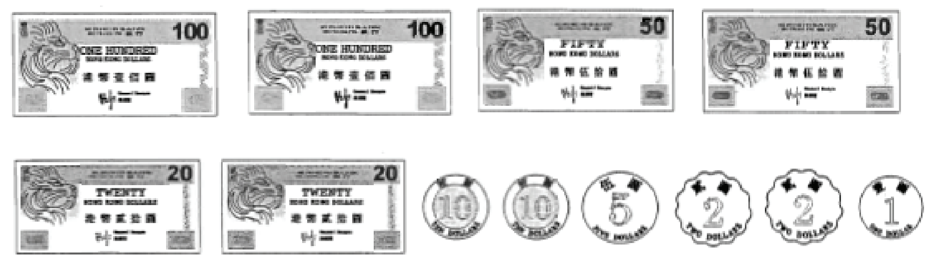
(b) There were 11 days of holidays for Christmas and the New Year. The holidays began on the 23rd of December. After the holidays, the pupils went back to school again on the
_______ of ____________ , _______ .
(month) (year)
</td></tr>
<tr><td>Item Format</td><td></td><td>Short closed constructed-response</td></tr>
<tr><td>Computational Difficulty</td><td></td><td>Low (counting to 11)</td></tr>
<tr><td>Cognitive Complexity</td><td></td><td>Level 2 (identify the date of a day identified as “next Wednesday) with a partial calendar AND identify the date of a day immediately following 11 days of holiday.</td></tr>
<tr><td>Comments</td><td></td><td>Two practical calendar problems</td></tr>
</table>

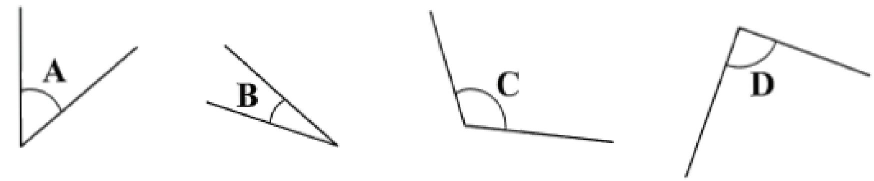
<table>
<tr><th colspan="3">Topic: Measurement—Money</th></tr>
<tr><th></th><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>Item</td><td>The table below shows how many coins each child in the Jones family has.

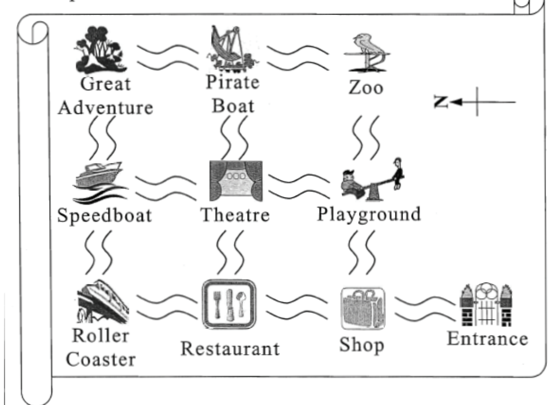
Children's Coins

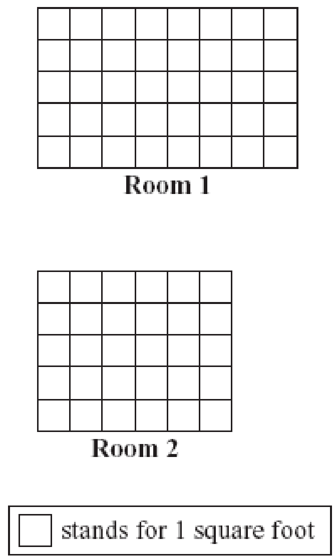
<table><tr><th>Child</th><th>Coins</th></tr><tr><td>Byron</td><td>3 quarters</td></tr><tr><td>Pam</td><td>100 pennies</td></tr><tr><td>Linda</td><td>4 quarters</td></tr><tr><td>William</td><td>50 pennies</td></tr></table>
Which two children have the same amount of money?

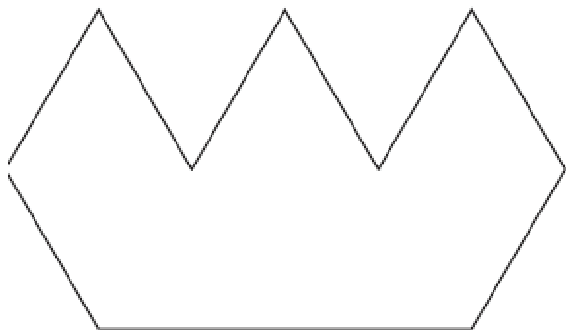
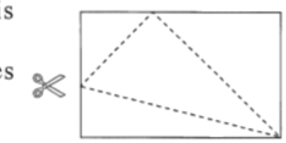
a. Linda and William
b. Pam and William
c. Byron and Pam
d. Pam and Linda</td><td>The shoe cabinet is on sale for three hundred and five dollars.

(a) Write the price in numerals on the price tag.

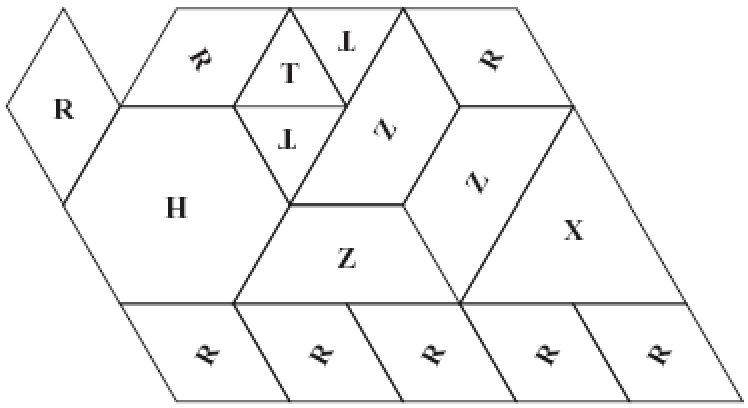
(b) Paul buys the shoe cabinet and pays with a  . How much change should he get?
Circle the amount of change.
</td></tr>
<tr><td>Item Format</td><td>Multiple choice</td><td>Short closed constructed-response</td></tr>
<tr><td>Computational Difficulty</td><td>Low (converts quarters and dimes to number of cents)</td><td>Medium (uses bills and coins to make change for $500 – $305)</td></tr>
<tr><td>Cognitive Complexity</td><td>Level 1 (identifying two greatest amounts of money presented as sets of coins)</td><td>Level 2 (makes change in problem situation)</td></tr>
<tr><td>Comments</td><td>Must know that there are 25 cents in 1 quarter.</td><td>Use of pictures facilitates counting-on strategy.</td></tr>
</table>

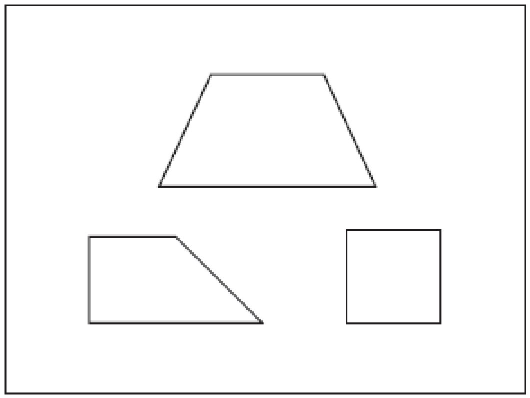
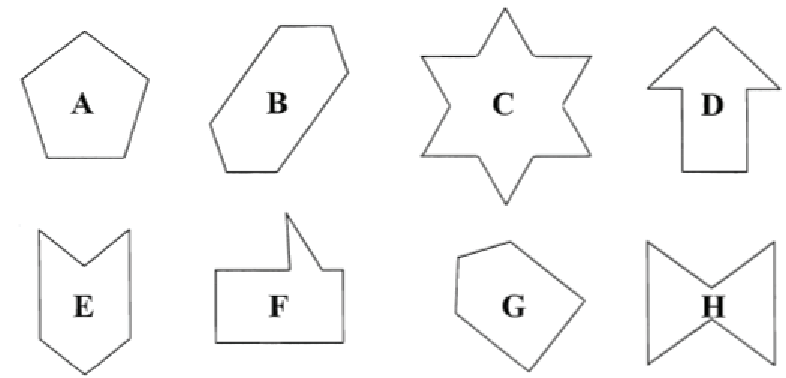
Topic: Measurement—Angles		
	Massachusetts	**Hong Kong**
Item	**THERE ARE NO ANGLE ITEMS ON THE MASSACHUSETTS TEST**	Arrange the angles in the diagram below in order, from the smallest to the largest.  Answer: ________ , ________ , ________ , ________ (Smallest) (Largest)
Item Format		Short closed constructed-response
Computational Difficulty		Low (no arithmetic)
Cognitive Complexity		Level 1 (order the size of four drawn angles from smallest to largest)
Comments		

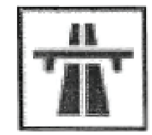
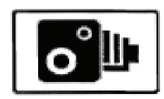
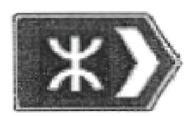
Topic: Measurement—Map/Directions		
	Massachusetts	**Hong Kong**
Item	**THERE ARE NO MAP/DIRECTIONS ITEMS ON THE MASSACHUSETTS TEST**	The map below shows the locations of the facilities in a theme park.  (a) After entering the theme park, Leo and his family go _________ (direction) all the way to the Roller Coaster. (b) The Zoo is to the _________ (direction) of the Shop. (c) The Pirate Boat is to the south of the _____________ . (d) After watching a performance at the Theatre, Leo and his family want to go to the Shop. They first go _________ (direction), pass the ______________ , and then go _________ (direction) to reach the Shop.
Item Format		Short closed constructed-response
Computational Difficulty		Low (no arithmetic)
Cognitive Complexity		Level 2 (requires application and translation of north, south, east, and west given an arrow and a map)
Comments		

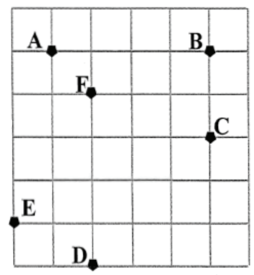
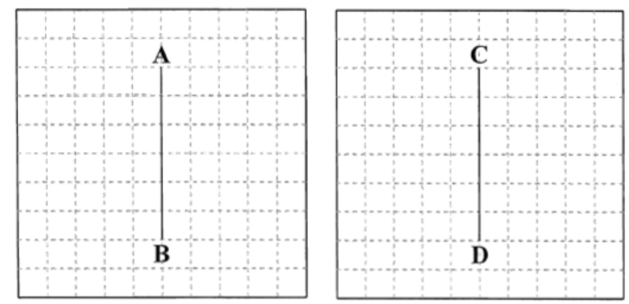
Topic: Measurement—Area		
	Massachusetts	**Hong Kong**
Item	Models of Room 1 and Room 2 are shown below. Each room is shaped like a rectangle.  How many square feet larger is the area of Room 1 than the area of Room 2? a. 2 square feet b. 4 square feet c. 8 square feet d. 10 square feet	**THERE ARE NO AREA ITEMS ON THE HONG KONG TEST**
Item Format	Multiple choice	
Computational Difficulty	Low (two multiplication facts and a subtraction)	
Cognitive Complexity	Level 1 (comparing 5 × 8 = 40 with 5 × 6 = 30)	
Comments		

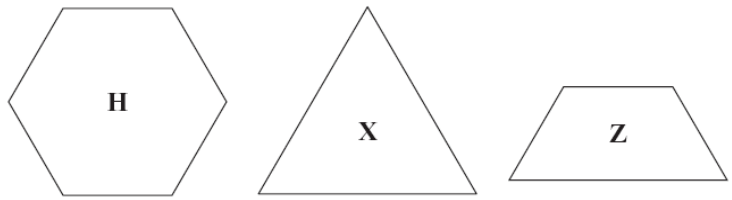
Topic: Geometry—2-D Shapes		
	Massachusetts	**Hong Kong**
Item	Use shapes R, T and Z from your tool kit to answer this question. [see next page] Which set of four shapes could you use to make the figure below?  a. R, R, T, Z b. R, R, R, Z c. R, R, Z, Z d. R, R, T, T	The rectangle shown on the right is cut into four triangles along the dotted lines. What type of triangles are these? Answer: ______________ triangle 
Item Format	Multiple choice	Short closed constructed-response
Computational Difficulty	Low (no arithmetic)	Low (no arithmetic)
Cognitive Complexity	Level 2 (spatial visualization identifying which four shapes “cover” a given figure)	Level 1 (identify three triangles as right triangles)
Comments		The three triangles including corners of the rectangle are certainly right triangles, but the interior triangle is right only if the top right and top left triangles are isosceles.

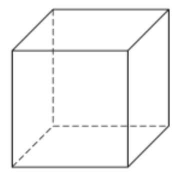
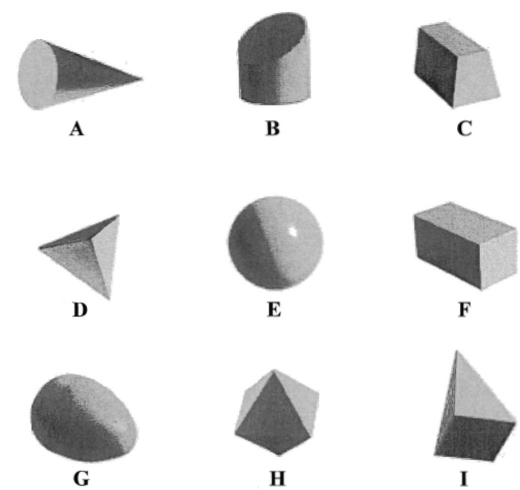

R
R
T
T
T
R
H
Z
Z
Z
X
R
R
R
R
R

Topic: Geometry—2-D Shapes		
	Massachusetts	**Hong Kong**
Item	Anna has the shapes shown in the box below. **Anna's Shapes**  Which statement best describes Anna's shapes? a. Each shape has all sides of equal length. b. Each shape has one line of symmetry. c. Each shape has four right angles. d. Each shape has four corners.	Which of the following 2-D shapes are hexagons? Write the letters for the answer.  Answer: ____________________
Item Format	Multiple choice	Short closed constructed-response
Computational Difficulty	Low (no arithmetic)	Low (no arithmetic)
Cognitive Complexity	Level 1 (recognize that each shape is a quadrilateral)	Level 1 (identify which three of eight shapes has six sides)
Comments		

Topic: Geometry—2-D Shapes		
	Massachusetts	**Hong Kong**
Item		The following are some road signs. According to the shape of the road signs, what types of 2-D shapes are they? (a)  Answer: ________ (b)  Answer: ________ (c)  Answer: ________ (d)  Answer: ________ (e)  Answer: ________
Item Format		Short closed constructed-response
Computational Difficulty		Low (no arithmetic)
Cognitive Complexity		Level 1 (identifying shapes as triangles, rectangles, squares, circles, and pentagons)
Comments		

Topic: Geometry—2-D Shapes			
	Massachusetts	Hong Kong	
Item		 Martin wants to make a right-angled triangle on the abov pin-board using an elastic band. Which points on the pin-board should he choose? Write the letters for the answer. Answer: ______________________	(a) On **Diagram 1**, draw a straight line that is parallel to AB.  **Diagram 1** **Diagram 2** (b) On **Diagram 2**, draw a straight line that is perpendicular to CD.
Item Format		Short closed constructed-response	Short closed constructed-response
Computational Difficulty		Low (no arithmetic)	Low (no arithmetic)
Cognitive Complexity		Level 1 (must identify two pairs of points or segments that are perpendicular)	Level 1 (draw a line parallel to and a line perpendicular to a given line segment)
Comments			

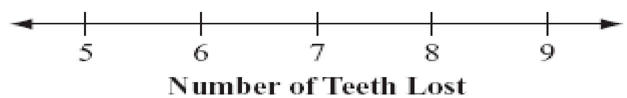
Topic: Geometry—Symmetry		
	Massachusetts	**Hong Kong**
Item	*Use shapes* H, X, *and* Z *from your tool kit to answer question 35.* **35** Ravi is using the shapes shown below to learn about symmetry.  a. In the space below, trace the shape that has **only one** line of symmetry. b. Draw the line of symmetry onto the shape you traced in part (a).	**THERE ARE NO SYMMETRY ITEMS ON THE HONG KONG TEST**
Item Format	Short closed constructed-response	
Computational Difficulty	Low (no arithmetic)	
Cognitive Complexity	Level 2 (identify a figure with only one line of symmetry and then draw that line)	
Comments		

Topic: Geometry—3-D Shapes		
	Massachusetts	**Hong Kong**
Item	A cube is shown below.  How many corners does a cube have? Write your answer in the Answer Box below.	Study the following 3-D shapes. Write the letter(s) for the answer.  (a) Pyramid(s): ____ (b) Cone(s): ____ (c) Prism(s): ____ (d) Sphere(s): ____
Item Format	Short closed constructed-response	Short closed constructed-response
Computational Difficulty	Low (no arithmetic)	Low (no arithmetic)
Cognitive Complexity	Level 1 (identify that a cube has eight corners or count based on an understanding of “corner”)	Level 2 (classify a set of nine figures as pyramids, cones, prisms, and spheres, or none of these)
Comments		Includes shapes (B and G) that are named.

Topic: Geometry—3-D Shapes		
	Massachusetts	**Hong Kong**
Item		Charles stacks up seven $1 coins. What kind of 3-D shape is the stack of coins?  ◯ A. Pyramid ◯ B. Cylinder ◯ C. Prism ◯ D. Sphere
Item Format		Multiple choice
Computational Difficulty		Low (no arithmetic)
Cognitive Complexity		Level 1 (identify a stack of coins as a cylinder)
Comments		

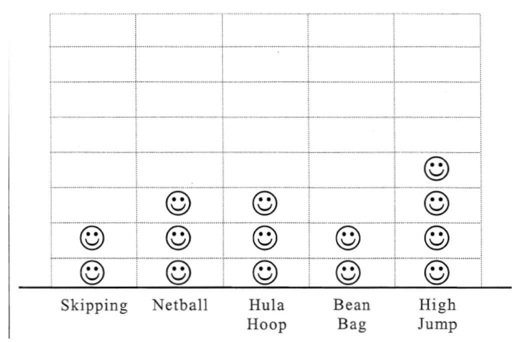
<table>
<tr><th colspan="3">Topic: Data—Constructing Graphs</th></tr>
<tr><th></th><th>Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>Item</td><td>Noah asked eight of his friends, “How many teeth have you lost?” The chart shows their answers.

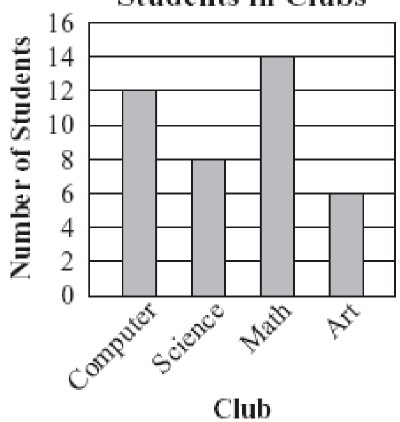
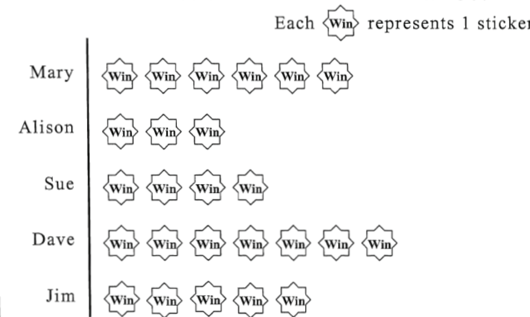
Number of Teeth Lost
<table>
<tr><th>Name of Friend</th><th>Number of Teeth Lost</th></tr>
<tr><td>Alex</td><td>6</td></tr>
<tr><td>Jennifer</td><td>8</td></tr>
<tr><td>Kyle</td><td>7</td></tr>
<tr><td>Cody</td><td>5</td></tr>
<tr><td>Amanda</td><td>8</td></tr>
<tr><td>Sammy</td><td>7</td></tr>
<tr><td>Dan</td><td>9</td></tr>
<tr><td>Sarah</td><td>8</td></tr>
</table>

Noah started to make the line plot below to show the data from his chart. Put Xs above the correct numbers to complete the line plot.

</td>
<td>Miss Wong did a survey on the participation of P.3B pupils in the activities for Games Day.

(a) According to the records, complete the table below.
<table>
<tr><th>Activities</th><th>Skipping</th><th>Netball</th><th>Hula Hoop</th><th>Bean Bag</th><th>High Jump</th></tr>
<tr><td>Records</td><td>卌 |</td><td>|||</td><td>卌</td><td>||||</td><td>卌 ||</td></tr>
<tr><td>Number of pupils</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

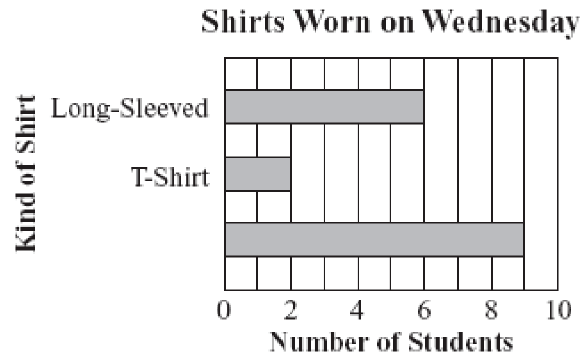
(b) According to the above data, complete the pictogram below.

The Participation of P.3B Pupils on Games Day
Each ☺ represents 1 pupil

</td></tr>
<tr><td>Item Format</td><td>Short closed constructed-response</td><td>Short closed constructed-response</td></tr>
<tr><td>Computational Difficulty</td><td>Low (counting 1 to 9)</td><td>Low (counting 1 to 7)</td></tr>
<tr><td>Cognitive Complexity</td><td>Level 1 (a multistep item requiring students to convert the data in a table to a line plot that is already set up.</td><td>Level 1 (a multistep item requiring students to convert the tallies in a table to numbers and then convert the data in the table to a simple 1-to-1 picture graph that is already set up and partially completed)</td></tr>
<tr><td>Comments</td><td>Engaging and appropriate context, one-digit numbers.</td><td>Engaging and appropriate context, one-digit numbers.</td></tr>
</table>

Topic: Data—Interpreting Graphs		
	Massachusetts	**Hong Kong**
Item	The bar graph below shows the number of students who belong to each club at Patterson School. **Students in Clubs**  How many **more** students belong to the Math Club than to the Art Club? a. 4 b. 6 c. 8 d. 14	Five children went to a fun fair. A sticker was given to each winner of a game. The pictogram below shows the number of stickers they got. **Number of Stickers the Five Children Got**  (a) The child who won the greatest number of games got ________ more stickers than the child who won the least number of games. (b) The children who got 6 stickers or more were given a gift. According to the pictogram above, the child/children who got a gift was/were ____________________________ . (c) If each child could play two more games, the child/children who still had a chance of getting a gift was/were ____________________________ .
Item Format	Multiple choice	Short closed constructed-response
Computational Difficulty	Low (subtract 14 – 8)	Low (order, sums and differences with numbers 1 to 10)
Cognitive Complexity	Level 2 (reading two numbers from a bar graph and subtracting them)	Level 2+ (A three-part item requiring students to interpret a 1-to-1 picture graph involving 1) finding the difference between the greatest and least, 2) identifying who has 6 or more, and 3) recognizing that with two more games, two additional children could get to 6 –which adds nuance.
Comments	Reasonable context and simple graph	Heavy reading load

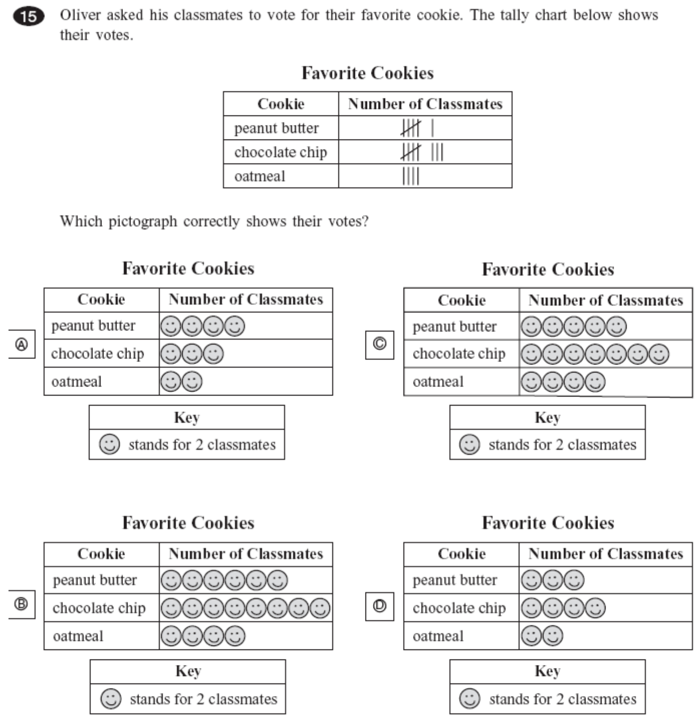
<table>
<tr><th colspan="4">Topic: Data—Interpreting Graphs</th></tr>
<tr><th></th><th colspan="2">Massachusetts</th><th>Hong Kong</th></tr>
<tr><td>Item</td><td>The tally chart below shows the favorite lunches of some students.

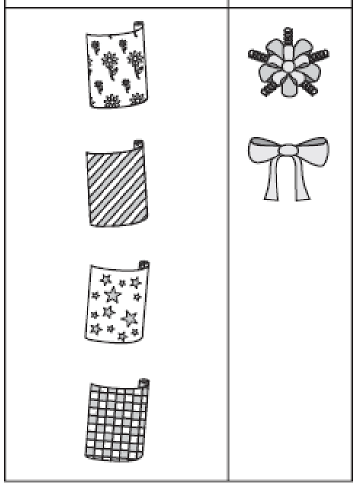
Favorite Lunches
<table><tr><th>Lunch</th><th>Number of Students</th></tr><tr><td>Taco</td><td>卌</td></tr><tr><td>Chicken</td><td>|||</td></tr><tr><td>Spaghetti</td><td>||||</td></tr><tr><td>Hamburger</td><td>卌</td></tr><tr><td>Pizza</td><td>卌 |</td></tr></table>
Which two lunches were the favorites of the same number of students?
a. taco and spaghetti
b. taco and hamburger
c. spaghetti and hamburger
d. spaghetti and pizza</td><td>Anne made the graph below to show the kinds of shirts worn by her classmates on Wednesday.



What information is missing from Anne's graph?
a. the kind of shirt worn by 9 students
b. the kind of shirt worn by 2 students
c. the number of students who wore T-shirts
d. the number of students who wore long-sleeved shirts</td><td></td></tr>
<tr><td>Item Format</td><td>Multiple choice</td><td>Multiple choice</td><td></td></tr>
<tr><td>Computational Difficulty</td><td>Low (no computational)</td><td>Low (no computational)</td><td></td></tr>
<tr><td>Cognitive Complexity</td><td>Level 1 (identifies two tallies as equivalent)</td><td>Level 1 (identify missing label)</td><td></td></tr>
<tr><td>Comments</td><td></td><td></td><td></td></tr>
</table>

Topic: Data—Equivalent Graphs

	Massachusetts	Hong Kong
Item		
Item Format	Multiple choice	
Computational Difficulty	Low (divide by 2)	
Cognitive Complexity	Level 2 (converting tallies to pictures, where picture = 2)	
Comments	Distractor B is likely to draw many students who see 1:1 tally to smiley face correspondence.	

Topic: Data—Combinations		
	Massachusetts	**Hong Kong**
Item	Mandy is going to wrap a gift. The kinds of wrapping paper and bows she can choose are shown below. **Gift Wrap** Kinds of Wrapping Paper / Kinds of Bows  How many different ways can Mandy choose 1 kind of wrapping paper and 1 kind of bow? a. 2 b. 4 c. 6 d. 8	**THERE ARE NO COMBINATIONS ITEMS ON THE HONG KONG TEST**
Item Format	Multiple choice	
Computational Difficulty	Low (multiplying 4 × 2)	
Cognitive Complexity	Level 2 (find the number of combinations in a problem situation)	
Comments		

Topic: Algebra—Patterns			
	Massachusetts		Hong Kong
Item	Zoey is using bananas and oranges to make the pattern shown below. The rule for her pattern is ABBB.  Zoey will follow the rule for her pattern a total of 4 times. How many **oranges** will Zoey use in all? Show or explain how you got your answer.	Ms. Mackey wrote the number pattern below using the rule "subtract 8." 187, 179, 171, __?__, 155, 147, 139 What is the missing number in Ms. Mackey's pattern? a. 163 b. 168 c. 170 d. 177	**THERE ARE NO PATTERNS ITEMS ON THE HONG KONG TEST**
Item Format	Open constructed response	Multiple choice	
Arithmetic difficulty	Low (multiply 3 × 4)	Low (identify pattern as "subtract 8" and apply it to 171)	
Cognitive Complexity	Level 1 (repeat a pattern and either count or multiply to find the answer)	Level 2 (two steps with three-digit numbers)	
Comments	This requires "show or explain" and is one of only two open constructed response items.	Subtraction is embedded in a pattern item requiring finding the missing number in a "subtract 8" pattern. May involve regrouping.	

Topic: Algebra—Number Sentences			
	Massachusetts		**Hong Kong**
Item	Which number sentence is true? Ⓐ $5 + 0 = 5 \times 1$ Ⓑ $5 + 1 = 5 \times 1$ Ⓒ $5 + 0 = 5 \times 0$ Ⓓ $5 + 1 = 5 \times 0$	Candace wrote the number sentence below. $15 \div 3 = \square$ Which of these is another way to write Candace's number sentence? Ⓐ $15 + \square = 3$ Ⓑ $15 \times \square = 3$ Ⓒ $3 + \square = 15$ Ⓓ $3 \times \square = 15$	**THERE ARE NO NUMBER SENTENCE ITEMS ON THE HONG KONG TEST**
Item Format	Multiple choice	Multiple choice	
Computational Difficulty	Low (basic facts)	Low (basic facts)	
Cognitive Complexity	Level 1 (identify sentence that shows that adding zero and multiplying by 1 are identities)	Level 1 (identify number sentence that shows that division and multiplication are inverse operations)	
Comments			